An Interactive Decision-making Method for Autonomous Valet Parking Based on Non-cooperative Complete Information Static Game

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***Abstract*—For the decision-making problem involving the interaction of an autonomous vehicle with other participating vehicles during the process of autonomous valet parking (AVP), an interactive decision-making method is proposed based on non-cooperative complete information static game theory. Firstly, the conflict types that occur between participating vehicles during the AVP process are analyzed, specifically in the intersection scenario and the front of the parking spot scenario within the parking lot. And all conflict modes are systematically summarized based on the inductive method. Then, the non-cooperative complete information static game is adopted to model the vehicle-vehicle interaction process, additionally, an interactive game cost function is designed by integrating driving style, driving safety, and traffic efficiency. Finally, the optimal driving strategy is obtained by solving the Nash equilibrium solution of the non-cooperative complete information static game. Through simulating various diverse vehicle-vehicle interaction scenarios in the parking lot, the experimental results demonstrate that this method dynamically determines correct and reasonable driving strategies and the effectiveness of the proposed method is verified.**

Keywords—autonomous valet parking, interactive decision-making, vehicle-vehicle interaction, non-cooperative complete information static game

I. INTRODUCTION

The AVP system takes full control of the vehicle without the need for driver operation or supervision, enabling fully automated driving between the parking lot exit and the parking spot[1]. But the interaction between the ego vehicle and the participating vehicle in the process of AVP is very common. For the ego vehicle to make reasonable decisions, it is necessary to consider the influence of the interaction between the ego vehicle and the participating vehicle.

As an analysis theory, game theory is valuable in describing the process of interactive decision-making and obtaining the optimal strategy[2][3]. In the study of vehicle interaction using game theory, Kita et al. first considered the interaction between the on-ramp merging vehicle and the straight-through vehicle. They elucidated the interaction process between the ego vehicle and other vehicles during merging using the game theory[4]. Since then, a large number of studies have focused on lane change or car-following scenarios, but research on conflicts in intersection scenarios is relatively limited. Fox et al. gridded the intersection and constructed a cost function based on the distance from the traffic participant to the central grid. The discrete sequential game theory was adopted to conduct a decision of the ego vehicle at the unsigned intersection[5]. However, the grid resolution is difficult to define, and the incomplete of the cost function also makes the Nash equilibrium solution unreasonable. Rahmati et al. utilized the information flow within a networked environment to capture the dynamic interaction behavior between a vehicle and other vehicles during turning left. They developed a model using a two-person non-zero sum non-cooperative game under complete information to address the interaction problem[6]. Hang et al. considered personalized driving preferences, integrated driving safety, traffic efficiency, and driving aggressiveness to construct a cost function. They then applied differential games to find two equilibrium solutions, which solved the driving conflict at unsignalized intersections[7]. About driving style, Elvik analyzed the hawk-dove driver style and confirmed that when two dove drivers encounter each other, they can benefit from cautious behavior. However, the change in the driving style of the other vehicle during the interaction process influences the driving style of the ego vehicle, leading it to lean towards either hawk-style or dove-style driving to achieve optimal benefits[8]. In vehicle longitudinal control, the eagle-dove driving style is often categorized based on vehicle speed and acceleration [9], which provides an intuitive assessment of driving aggressiveness. Driving styles are typically classified into three types: conservative driving, normal driving, and aggressive driving[10][11]. It can be observed that existing interaction models do not cover various interaction scenarios in the parking lot, rendering the conflict model incomplete. Furthermore, there is a lack of multi-metric cost functions considering driving style for interactive decision-making applicable in AVP.

Given the aforementioned problems, an interactive decision-making method for AVP is proposed, taking into account the hawk-dove driver style, and based on the non-cooperative complete information static game theory. The proposed method comprehensively considers driver style, driving safety, and traffic efficiency, and the cost function is formulated to capture different behavior combinations in the interaction process between the ego vehicle and participating vehicles, then the reasonable decision-making result is

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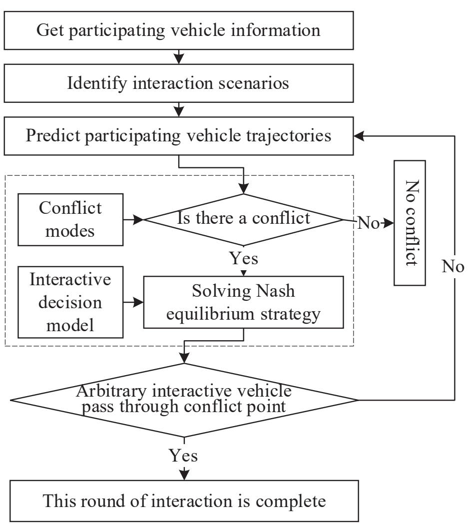


Fig. 2. Vehicle-vehicle interaction decision-making framework during the AVP process

obtained by solving the Nash equilibrium solution. Meanwhile, the conflict modes of vehicle-vehicle interaction are comprehensively analyzed based on the induction method, and various forms of interaction are fully verified.

II. Interactive Decision-making Method

A. Methodological Framework

For addressing the decision-making problem of vehicle-vehicle interaction in the AVP process, a rule-based method is employed to establish the vehicle-vehicle interaction decision framework, as depicted in Fig. 1. Firstly, the pose and speed information of the participating vehicles are acquired according to the external perception data. By combining this information with the vehicle localization information, the vehicle-vehicle interaction scenario is identified in the high-precision map. Subsequently, the future trajectory of participating vehicle is predicted utilizing the state and prior map information. In conjunction with the pre-established conflict model, it is determined whether a conflict exists between the vehicles. If no conflict is present, no interaction is required. However, If a conflict arises, the Nash equilibrium solution is solved based on the interaction model and vehicle state information, resulting in the optimal single strategy for the ego vehicle within one round of the game. Finally, it is assessed whether either party has passed the conflict point. If one party has passed, the interaction ends, and if both parties have not passed, the next round of interaction continues.

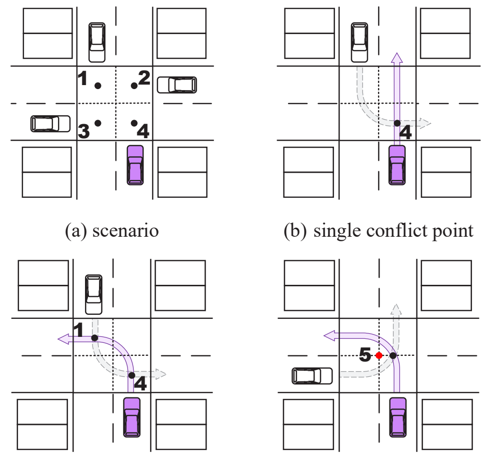
In this framework, the interactive decision-making method encompasses three main components: constructing conflict modes, establishing the interactive decision model, and solving the Nash equilibrium strategy. These components work together to facilitate effective decision-making in the context of vehicle-vehicle interaction.

B. Vehicle-vehicle Conflict Analysis

Based on the inductive method[12], the interactive scenarios in the parking lot are intersections and parking spots.

1) Vehicle-vehicle Conflict at Intersection

Conflict points refer to the intersection points where the ego vehicle and the traffic participant may collide during the



(a) scenario (b) single conflict point

(c) double conflict point (d) simplified conflict point

Fig. 1. Vehicle-vehicle interaction scenario at the intersection

interaction process. In the depicted scenario shown in Fig. 2(a), the intersection is divided into four blocks based on the road line markings. Conflict points are set at the center of blocks, the colored vehicle in the figure represents the ego vehicle, while the uncolored vehicle represents the participating vehicle. Conflicts arise when participating vehicles from other directions navigate through the intersection simultaneously with the ego vehicle. Vehicles exhibit three behaviors: going straight, turning left, and turning right. Therefore, according to the number and specificity of conflict points, the interaction can be categorized into three situations: single conflict point, double conflict point, and simplified conflict point.

For the situation of a single conflict point in Fig 2(b), when the ego vehicle goes straight and the opposite vehicle turns left, the intersection of the expected travel paths of vehicles is the No. 4 conflict point. This indicates that the vehicle occupying conflict point 4 first has the priority to pass through the intersection. In the case of the double conflict point depicted in Fig 2(c), when both the ego vehicle and the opposite vehicle turn left, they will pass through conflict points No. 1 and No. 4 at the intersection. Therefore, the No. 4 conflict point is designed for the ego vehicle, and the No. 1 conflict point is designed for the opposite vehicle. This indicates that when one of the vehicles occupies the assigned conflict point, it obstructs the anticipated driving path of the other vehicle. As a result, the vehicle that occupies its designated conflict point first is granted the priority to pass through the intersection. In the simplified conflict point situation depicted in Fig 2(d), when the ego vehicle and the left participating vehicle both intend to turn left, the conventional four conflict points no longer align with the actual situation. For ease of calculation, the principle of proximity is adopted, and the No.5 conflict point is simplified to the center of the intersection. Finally, all the behaviors during vehicle-vehicle interaction are listed, and the conflict points are summarized as shown in Table I.

2) Vehicle-vehicle Conflict in Front of Parking Spots

In the process of AVP, the vehicle not only pass through the intersection but also pass through parking spots. There is a high probability of interacting with exiting vehicles. When an exiting vehicle leaves the parking spot, there is a behavior of going forward and then turning right, and there is also a behavior of crossing the lane and turning left into the opposite lane. For the above different ways of exiting the parking spots, interactive conflict points in front of parking spots are set at

TABLE I. CONFLICT POINTS AT INTERSECTIONS

Participating Vehicle Behavior	Ego vehicle Behavior		
	going straight	*turning left*	*turning right*
in the opposite direction and going straight	-	1	-
in the opposite direction and turning left	4	1,4	4
in the opposite direction and turning right	-	1	-
on the right and going straight	2	1	-
on the right and turning left	2	5	-
on the right and turning right	2	-	-
on the left and going straight	4	4	4
on the left and turning left	2	5	-
on the left and turning right	-	-	-

the center points of the intersection area between the center lane of the ego vehicle lane and the front of the parking spots where the interactive vehicle is located, as shown in Fig. 3.

C. Interactive Decision Modeling

When the vehicle approaches an intersection or the area in front of the parking spot, the presence of other participating vehicles will influence the decision-making process of the ego vehicle. Both of them continuously seek to optimize their interests during the interaction. Therefore, the entire interaction process can be viewed as a game. It is important to note that interactive players are unable to transfer coordination strategies or form alliances during the game. Thus, the interaction process is defined as a non-cooperative complete information static game[13][14]. In each game round, the following definitions are established:

$$G = \left(S_i, u_i\right)_{i=1}^{N} \tag{1}$$

Among them, the number of participating vehicles in the game is N, the strategy set of vehicle i is $\mathrm{S}_i = \{s_{i,1}, s_{i,2}, \ldots, s_{i,M}\}$, each strategy set contains M pure strategies, and the cost function income $u_i \in \mathbb{R}$ is calculated from the strategies of each vehicle, then the Nash equilibrium solution $s^* \in \mathrm{S}$ of the non-cooperative game model can be obtained. For the two-vehicle interactive game problem, the strategy set S_i of vehicle i is defined as follows:

$$\mathrm{S}_i = \left\{s_{i,\mathrm{y}}, s_{i,\mathrm{n}}, s_{i,\mathrm{s}}\right\} \quad , \quad i \in [1,2] \tag{2}$$

Among them, i = 1 is defined as an automated driving vehicle, i = 2 is defined as a participating vehicle, $s_{i,y}$ represents stop-and-wait, $s_{i,n}$ represents driving at a normal speed, and $s_{i,s}$ represents driving at a slow speed. Construct a cost function for the above strategy:

$$u_i = \mathrm{U} - c_{\mathrm{d},i} \cdot c_{\mathrm{s},i} \cdot f_{\mathrm{e},i} \quad , \quad i \in [1,2] \tag{3}$$

In (3), the cost function u_i is determined based on the constant U, the vehicle-vehicle interaction eagle-dove driving style factor $c_{d,i}$, the driving safety factor $c_{s,i}$, and the traffic efficiency function $f_{e,i}$. The term $f_{e,i}$ is the ratio of the time t_i of each vehicle to the conflict point and the time $t_{1,\mathrm{n}}$ of the autonomous vehicle to the conflict point at a normal speed, as shown in (4). The constant speed model is used to estimate the passing time, so the lower the traffic efficiency, the higher the cost value u_i of the vehicle under this strategy.

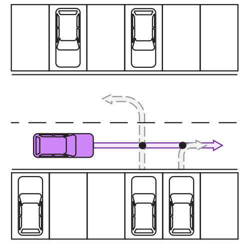


Fig. 3. Vehicle-vehicle interaction scenario in front of parking spots

$$f_{\mathrm{e},i} = \frac{t_i}{t_{1,\mathrm{n}}} \quad , i \in [1,2] \tag{4}$$

Considering that the eagle-dove driving style factor is related to vehicle speed, this factor only acts on the normal-speed and slow-speed strategies, and it is assigned a constant 1 in the stop-and-wait strategy, as shown in (5). The acceleration a_{-i} of the participating vehicle is used as the basis for judging whether the driving style of the participating vehicle is converted. And the driving style factor $c_{d,i,k} \in \{1/\omega_\mathrm{s}^2, 1/\omega_\mathrm{d}, 1, \omega_\mathrm{d}, \omega_\mathrm{s}^2\}$ of the ego vehicle is divided into 5 driving style levels. When the calculation result is greater than the boundary value, it remains at the boundary value, the value is 1 at the initial moment $k = 0$.

$$c_{\mathrm{d},i,k} = \begin{cases} c_{\mathrm{d},i,k-1} & , \ a_{-i} = 0 \\ 1/\omega_\mathrm{d} \cdot c_{\mathrm{d},i,k-1} & , \ a_{-i} < 0 \\ \omega_\mathrm{d} \cdot c_{\mathrm{d},i,k-1} & , \ a_{-i} > 0 \end{cases} \tag{5}$$

The driving safety factor is related to the vehicle speed, so this factor only acts on the normal-speed and slow-speed strategies, and it is assigned a constant 1 in the stop-and-wait strategy, as shown in (6). Based on the time ratio $\eta = t_i / t_{-i}$ between the ego vehicle and the participating vehicle reaching the conflict point, a total of 4 thresholds $\{T_1, T_2, T_3, T_4\}$ are set, and 5 action intervals are divided. Different safety factors $\{\omega_\mathrm{s}, 1, 1/\omega_\mathrm{s}^2\}$ are utilized in different intervals. For the normal-speed strategy and the slow-speed strategy, the driving safety factors are defined as $c_{\mathrm{sn},i}$ and $c_{\mathrm{ss},i}$ respectively.

$$c_{\mathrm{s},i} = \begin{cases} \omega_\mathrm{s} & T_2 \le \eta \le T_3 \\ 1 & T_1 \le \eta < T_2 \text{ 或 } T_3 < \eta \le T_4 \\ 1/\omega_\mathrm{s}^2 & \eta < T_1 \text{ 或 } \eta > T_4 \end{cases} \tag{6}$$

The safe traffic efficiency of each strategy in the vehicle-vehicle interaction is shown in Table II. When both vehicles choose stop-and-wait, it is equivalent to the time for the two vehicles to reach the conflict point being infinite, so the cost of the two vehicles under this strategy is ∞. When the participating vehicle chooses stop-and-wait, and the autonomous vehicle chooses to pass at a normal speed, the passing time of the autonomous vehicle is $t_{1,\mathrm{n}}$, and the participating vehicle waits for the autonomous vehicle to pass before passing, so the passing time is $t_{1,\mathrm{n}} + t_{1,\mathrm{n}}$. When both choose to pass at a normal speed, the passing time of the autonomous vehicle is $t_{1,\mathrm{n}}$, and the passing time of the participating vehicle is $t_{1,\mathrm{n}}$.

D. Pure Strategy Solution of Nash Equilibrium

The solution of Nash equilibrium is divided into pure strategy solution and mixed strategy solution. In this research, the pure strategy solution of Nash equilibrium is adopted as

TABLE II. SAFETY TRAFFIC EFFICIENCY DURING VEHICLE-VEHICLE INTERACTION

Participating vehicle behavior	*Autonomous vehicle behavior*	*Participating vehicle safety and traffic efficiency*	*Autonomous vehicle safety and traffic efficiency*
stop-and-wait	stop-and-wait	∞	∞
stop-and-wait	normal-speed	$\left(t_{1,n}+t_{2,n}\right)/t_{1,n}$	$c_{d,1}\cdot c_{sn,1}\cdot t_{1,n}/t_{1,n}$
stop-and-wait	slow-speed	$\left(t_{1,s}+t_{2,n}\right)/t_{1,n}$	$c_{d,1}\cdot c_{ss,1}\cdot t_{1,s}/t_{1,n}$
normal-speed	stop-and-wait	$c_{d,2}\cdot c_{sn,2}\cdot t_{2,n}/t_{1,n}$	$\left(t_{2,n}+t_{1,n}\right)/t_{1,n}$
normal-speed	normal-speed	$c_{d,2}\cdot c_{sn,2}\cdot t_{2,n}/t_{1,n}$	$c_{d,1}\cdot c_{sn,1}\cdot t_{1,n}/t_{1,n}$
normal-speed	slow-speed	$c_{d,2}\cdot c_{sn,2}\cdot t_{2,n}/t_{1,n}$	$c_{d,1}\cdot c_{ss,1}\cdot t_{1,s}/t_{1,n}$
slow-speed	stop-and-wait	$c_{d,2}\cdot c_{ss,2}\cdot t_{2,s}/t_{1,n}$	$\left(t_{2,s}+t_{1,n}\right)/t_{1,n}$
slow-speed	normal-speed	$c_{d,2}\cdot c_{ss,2}\cdot t_{2,s}/t_{1,n}$	$c_{d,1}\cdot c_{sn,1}\cdot t_{1,n}/t_{1,n}$
slow-speed	slow-speed	$c_{d,2}\cdot c_{ss,2}\cdot t_{2,s}/t_{1,n}$	$c_{d,1}\cdot c_{ss,1}\cdot t_{1,s}/t_{1,n}$

the game result. For the finite strategy set, the classic underline method is used to solve Nash equilibrium. In the general game situation, it is not guaranteed that a pure strategy Nash equilibrium solution exists[15]. To avoid the possibility of no solution, a solution strategy is designed. When there is a solution, the Nash equilibrium solution is implemented. If no solution is found, the self-vehicle chooses either the stop-and-wait strategy or the slow-speed strategy based on the safest principle. The strategy selection is based on the ratio of the time t_1 that the ego vehicle arrives at the conflict point and the time t_2 that the participating vehicle arrives at the conflict point. If the ratio is greater than the safety factor λ, it indicates both vehicles are driving in a state with no safety risk, and the slow-speed strategy is chosen. But if the ratio is not greater than λ, it indicates that both vehicles are driving in a state with hazards, and the stop-and-wait strategy can only be chosen.

III. SIMULATION

A. Experimental Design

Based on the AVP software system[1], the proposed interactive decision-making module is introduced to carry out simulation experiments. For scenarios at the intersection and in front of parking spots, eight typical experiment conditions with different interaction forms were screened out from the conflict model to verify the proposed method. At the same time, vehicle interaction forms are categorized into three types: cross interaction, confluence interaction, and spiral interaction. The overall working condition design is shown in Table III.

The parameters of the interactive decision-making model are set as follows: the constant U is set to 10, ω_d is set to 3, ω_s is set to 3, and the interval division threshold $\{T_1, T_2, T_3, T_4\}$ of $c_{sn,i}$ under the normal-speed driving strategy is set to {0.8, 1, 2.5, 3}; under the slow-speed driving strategy, the interval division threshold of $c_{ss,i}$ is set to {0.8, 1, 1.5, 2}; the safety factor λ is set to 3. For the convenience of description, the ego vehicle (autonomous vehicle) that needs to make interactive decisions is abbreviated as AV (autonomous vehicle), and the participating vehicle is abbreviated as PV (participating vehicle). At the same time, the stop-and-wait strategy is defined as mode "1", the vehicle speed in this mode is 0 m/s; the normal-speed strategy is defined as mode "2", and the vehicle speed in this mode is 2 m/s; the slow-speed strategy is defined as mode "3" , the vehicle speed in this mode is 1 m/s.

TABLE III. VEHICLE-VEHICLE INTERACTION EXPERIMENT CONDITIONS

Condition number	*Scenario*	*Interactive form*	*AV behavior*	*PV behavior*
1	interaction	cross interaction	going straight	on the left and going straight
2	interaction	cross interaction	going straight	on the right and turning left
3	interaction	confluence interaction	turning left	on the right and going straight
4	interaction	confluence interaction	going straight	on the left and turning left
5	interaction	spiral interaction	turning left	on the right and turning left
6	interaction	spiral interaction	turning left	in the opposite direction and turning left
7	parking spots	confluence interaction	going straight	move forward and turning right
8	parking spots	cross interaction	going straight	move forward and turning left

B. Results Analysis

1) Vehicle-vehicle Conflict at Intersection

a) Cross interaction case

The two conditions of cross interaction involve the PV going straight through the intersection from the left road and the PV turning left from the right side and passing the intersection, as depicted in Fig. 4(a-1) and Fig. 4(b-1) respectively. In the first condition, the PV is intentionally set to maintain a normal speed without yielding to the AV, and the distance from AV to the conflict point is set to be less than a certain time distance from PV. As a result, the Nash equilibrium strategy for the AV is to consistently maintain a normal speed to pass through the intersection, as shown in Fig. 4(a-2). For the second condition, AV is positioned far from the conflict point, while the PV gradually accelerates from the slow-speed strategy to the normal-speed strategy. Initially, the AV strategy is stop-and-wait. Once the waiting distance is deemed sufficient, the Nash equilibrium strategy for the AV shifts from the stop-and-wait strategy to the slow-speed strategy, and eventually transitions to a normal speed, as shown in Fig. 4(b-2).

b) Confluence interaction case

The confluence interaction refers to a situation where one vehicle enters the expected lane of the other vehicle while passing through the intersection. Two experiment conditions are designed for this scenario: AV turns left while PV goes straight from the right, as depicted in Fig. 4(c-1). The ego vehicle goes straight while the other vehicle turns left from the left, as depicted in Fig. 4(d-1). For the first condition, the PV driving style and distance to the conflict point are similar to condition 2. The result of interactive decision-making is that the AV transitions from the conservative stop-and-wait strategy to normal-speed driving, as shown in Fig. 4(c-2). For the second condition, the dynamic change of the PV driving style from aggressive to conservative is set. At this time, the conservative style of AV leads to a Nash equilibrium solution of stop-and-wait. However, the Nash equilibrium solution changes due to the transformation of the PV strategy. As a result, the AV strategy transitions to the normal-speed strategy, as shown in Fig 4(d-2).

c) Spiral interaction case

The spiral interaction consists of two conditions: when the AV turns left and the left PV turns left, with the conflict point at the center of the intersection (the simplified conflict point);

when the AV turns left and the opposite PV turns left, resulting in the double conflict point, as depicted in Fig. 4(e-1) and 4(f-1) respectively. For the first condition, the initial time distance from the PV is similar to the time distance from the AV, the PV is set to have an aggressive driving style and enter the intersection at a normal speed. At this point, the AV adopts a stop-and-wait strategy as its initial response. Then transitions to the normal-speed strategy after reaching a certain time distance from the PV, as shown in Fig. 4(e-2). For the second condition, the PV driving style is set to dynamically change during the interaction. Initially, the PV starts to drive at a normal speed, but during the interaction, it suddenly changes from the stop-and-wait for preventive driving to aggressive. At this point, the AV strategy is shown in Fig 4(f-2). When the PV adopts a preventive stop-and-wait strategy, the AV strategy transitions from the stop-and-wait strategy to the normal-speed strategy. When the PV drives suddenly and violently, the Nash equilibrium moves again, and the AV strategy shifts to stop-and-wait.

2) Vehicle-vehicle Conflict in Front of Parking Spots

When the PV moves forward and turns right out of the parking spot without yielding to the AV, as depicted in Fig.

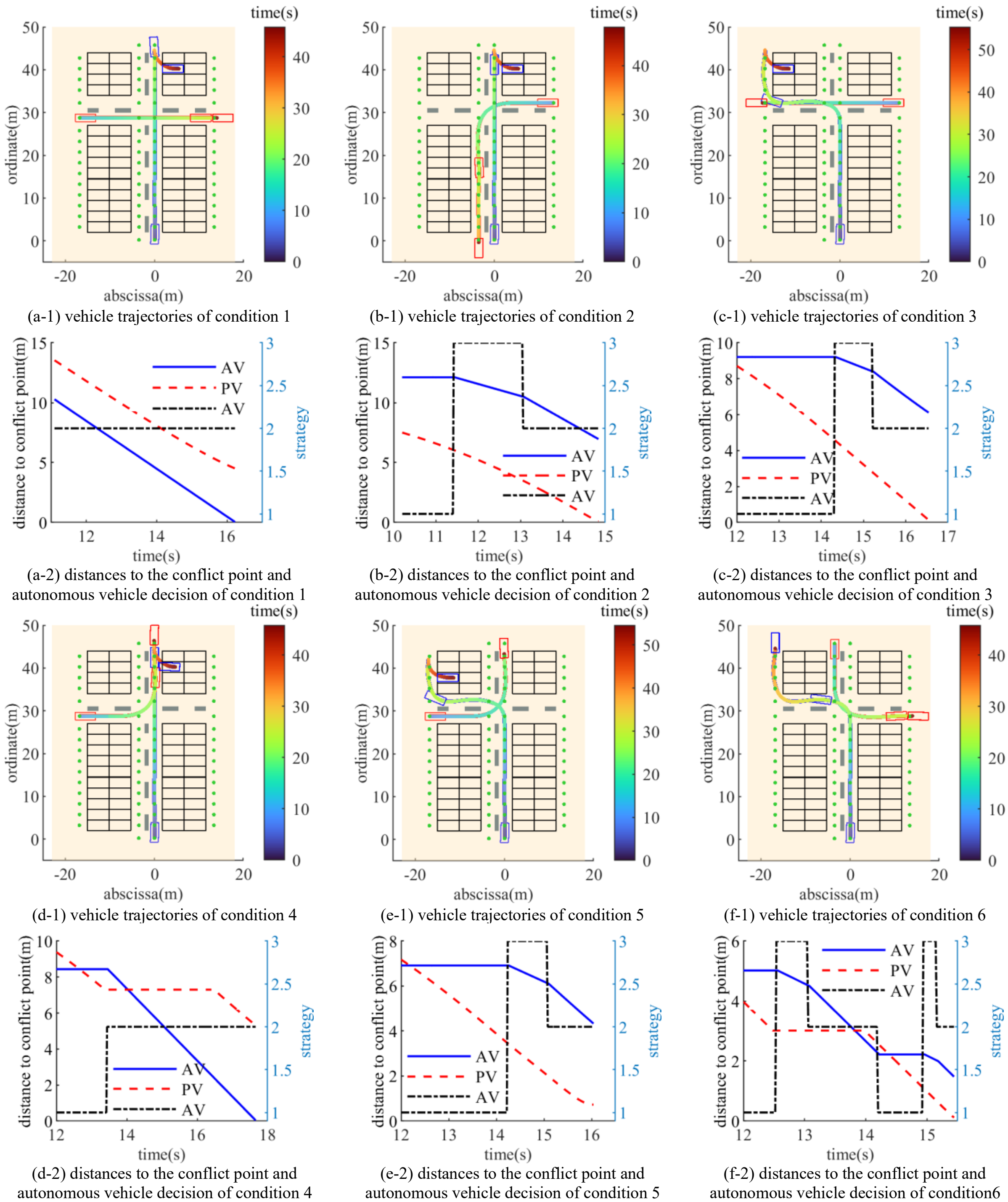
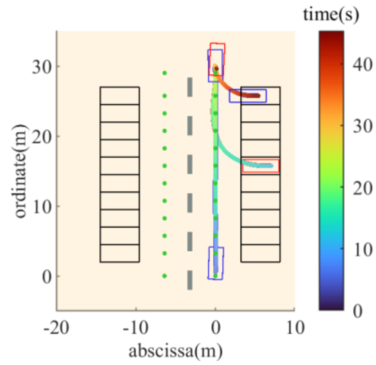


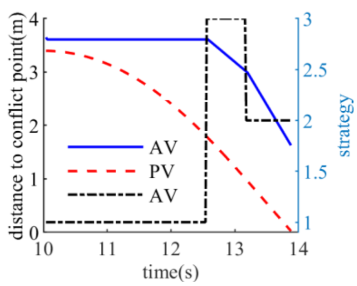
Fig. 4. Results of vehicle-vehicle conflict at the intersection

5(a-1). The AV adopts a yielding strategy when the distance from the AV to the conflict point is relatively long and the PV does not yield. This allows the PV to pass through the conflict point first. Once a sufficient safety distance is established, the AV transitions to driving at a slow speed and gradually returns to a normal driving speed, as shown in Fig. 5(a-2).

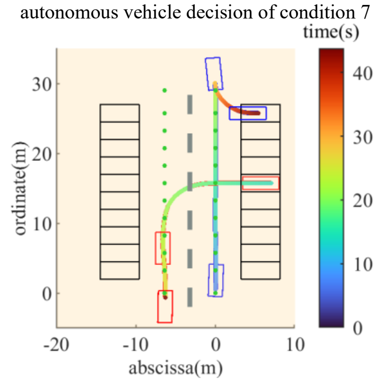
In the situation where the PV moves forward, turns left, exits the parking spot, and yields to the AV, as depicted in Fig. 5(b-1). PV leaves the parking spot, and its speed starts to increase from 0. The AV initially chooses stop-and-wait for conservative driving, while the PV conservatively chooses to stop-and-wait for politeness at the same time. According to the strategy of PV, the solution of AV gradually transitions to the slow-speed strategy and then returns to the normal-speed strategy, giving priority to passing through the conflict area. The entire interaction process is shown in Fig. 5(b-2).



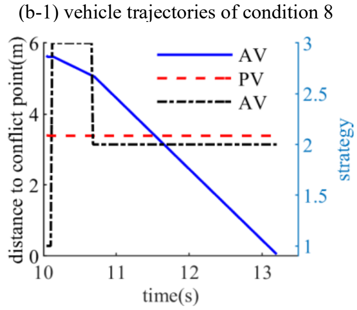
(a-1) vehicle trajectories of condition 7



(a-2) distances to the conflict point and autonomous vehicle decision of condition 7



(b-1) vehicle trajectories of condition 8



(b-2) distances to the conflict point and autonomous vehicle decision of condition 8

Fig. 5. Results of vehicle-vehicle conflict in front of parking spots

IV. Conclusion

(1) For various interaction situations in front of parking spots and intersections, the interactive decision-making method based on a non-cooperative complete information static game can provide reasonable decisions and has certain applicability. (2) This method fully considers various driving styles, driving safety, and traffic efficiency. Under the different driving styles of the participating vehicles, the effectiveness of dynamic decision-making as the behavior of traffic participants changes is fully verified.

In the future, the proposed method will be verified on the real vehicle.

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