Low-Complexity Optimization for Two-User Uplink NOMA Empowered by a Large Intelligent Reflecting Surface

Luiggi Cantos and Yun Hee Kim
Department of Electronics and Information Convergence Engineering
Kyung Hee University, Yongin 17104, Korea
E-mail: {lrcantos,yheekim}@khu.ac.kr

***Abstract*—We consider two-user uplink non-orthogonal multiple access (NOMA) assisted by a large intelligent reflecting surface (IRS). For the network, the rate fairness is maximized by optimizing the receive beamforming of a base station, power allocation of two users, and IRS reflection. For the two-user case, we derive the optimal power allocation (PA) and receive beamforming conditioned on IRS reflection in a closed form, which reduces the overall max-min rate optimization problem to the IRS reflection optimization. By approximating a nonsmooth minimum operation to a differentiable smooth function, we can solve the problem for a large number of IRS elements through a Quasi-Newton type nonlinear optimization algorithm that is known to be efficient for a large scale problem.**

***Index Terms*—Intelligent reflecting surface, non-orthogonal multiple access, nonlinear optimization, power allocation, receive beamforming**

I. Introduction

Intelligent reflecting surface (IRS) has emerged as a low cost and energy efficient method for 6G networks by changing channel environments smartly to meet various communication requirements [1]. Thus, various wireless communication networks have been reformed with IRS accompanied by IRS reflection optimization with the other resources. Among the networks, multiuser communications with non-orthogonal multiple access (NOMA) has been considered popularly due to its capability of higher spectral efficiency and more connectivity [2]–[7], [9].

Following the studies on the downlink IRS-NOMA, the uplink IRS-NOMA has been optimized to maximize the sum rate [4], [5] or the minimum rate [6], [7]. The authors considered a max-min rate optimization of the uplink IRS-NOMA with multiple receive antennas [7] which is accomplished by power allocation of the devices in addition to the optimization of receive beamforming and IRS reflection. The authors have proposed to apply a low-complexity nonlinear optimization algorithm called limited-memory Broyden-Fletcher-Goldfarb-Shanno bounded (L-BFGS-B) [8] to optimize power allocation and IRS reflection while adopting the conditionally optimal receive beamforming.

In this paper, we limit the problem considered for the uplink IRS-NOMA [7] to the case of a single NOMA pair; Two-user NOMA can be implemented with OMA such as time division multiple access in a hybrid form. For the problem, we present the objective as a function of power allocation and IRS reflection coefficients by using the conditionally optimal beamforming. We then derive the conditionally optimal power allocation as a function of IRS reflection coefficients, which renders the objective as a function of IRS reflection. Finally, the max-min rate optimization problem becomes solvable with the L-BFGS-B algorithm [8] by approximating the minimum of two variables with a smooth and differentiable function.

This research was supported by the National Research Foundation of Korea (NRF) under Grant NRF-2021R1A2C1005869 and by the Institute of Information & Communications Technology Planning & Evaluation (IITP) under the Information Technology Research Center (ITRC) support program IITP-2021-0-02046, with funding from the Ministry of Science and ICT (MSIT), Korea.

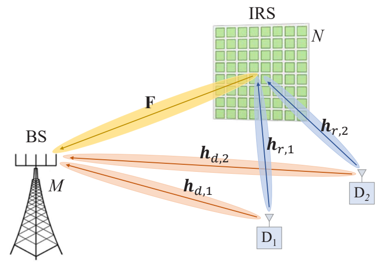

Fig. 1. Uplink IRS-assisted NOMA with two users.

Notation: $\mathbb{R}^{n\times m}$, $\mathbb{R}_+^{n\times m}$, and $\mathbb{C}^{n\times m}$ denote a set of $n\times m$ matrices with real values, positive real values, and complex values, respectively. We denoted by $[\mathbf{s}]_n$ the nth element of a vector $\mathbf{s}$, $\mathrm{diag}(\mathbf{s})$ the diagonal matrix with a diagonal vector $\mathbf{s}$, and $\mathcal{CN}(\boldsymbol{\mu},\boldsymbol{\Sigma})$ the complex Gaussian with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.

II. System Model and Problem Formulation

We consider an IRS-assisted uplink NOMA as depicted in Fig. 1, where a base station (BS) with M-antennas receives the signal from two single-antenna users with the help of an IRS with N-elements. Here, $\mathbf{F}\in\mathbb{C}^{M\times N}$ denotes the BS-IRS channel while $\mathbf{h}_{d,k}\in\mathbb{C}^{M\times 1}$ and $\mathbf{h}_{r,k}\in\mathbb{C}^{N\times 1}$ denote the channel from user k to the BS and IRS, respectively. Each user k simultaneously transmit its symbol s_k with $\mathbb{E}[|s_k|^2]=1$ at transmit power p_k for $k=1,2$. The received signal at the BS is expressed as

$$\boldsymbol{y}=\mathbf{h}_1(\boldsymbol{\phi})\sqrt{p_1}s_1+\mathbf{h}_2(\boldsymbol{\phi})\sqrt{p_2}s_2+\boldsymbol{n},\tag{1}$$

where $\boldsymbol{n}\sim\mathcal{CN}(\mathbf{0},\sigma^2\boldsymbol{I}_M)$ denotes the noise at the BS,

$$\mathbf{h}_k(\boldsymbol{\phi})=\mathbf{h}_{d,k}+\mathbf{F}\mathrm{diag}(\mathbf{h}_{r,k})\boldsymbol{\vartheta}(\boldsymbol{\phi})\tag{2}$$

is the equivalent channel from user k to the BS, and $\boldsymbol{\vartheta}(\boldsymbol{\phi})=[e^{j\phi_1},\cdots,e^{j\phi_N}]^T\in\mathbb{C}^{N\times 1}$ is the unit modulus IRS reflection vector. The BS detects symbol s_k by applying receive BF $\boldsymbol{w}_k$ with successive interference cancellation (SIC).

Without a loss of generality, we assume that user 1 experiences a stronger channel than user 2 as $\|\mathbf{h}_1(\boldsymbol{\phi})\|^2 \geq \|\mathbf{h}_2(\boldsymbol{\phi})\|^2$ for a given IRS reflection vector $\boldsymbol{\vartheta}(\boldsymbol{\phi})$ and a power allocation (PA) vector $\boldsymbol{p} = [p_1, p_2]^T$. The optimal receive beamforming is given by the minimum mean square error (MMSE) beamforming [9]

$$\boldsymbol{w}_k(\boldsymbol{p}, \boldsymbol{\phi}) = \boldsymbol{B}_k^{-1}(\boldsymbol{p}, \boldsymbol{\phi})\mathbf{h}_k(\boldsymbol{\phi}), \tag{3}$$

leading to the optimal SINR

$$\gamma_k(\boldsymbol{p}, \boldsymbol{\phi}) = p_k \boldsymbol{h}_k^H(\boldsymbol{\phi})\boldsymbol{B}_k^{-1}(\boldsymbol{p}, \boldsymbol{\phi})\boldsymbol{h}_k(\boldsymbol{\phi}), \tag{4}$$

where $\boldsymbol{B}_1(\boldsymbol{p}, \boldsymbol{\phi}) = \sigma^2 \boldsymbol{I}_M + p_2 \mathbf{h}_2(\boldsymbol{\phi})\mathbf{h}_2^H(\boldsymbol{\phi})$ and $\boldsymbol{B}_2 = \sigma^2 \boldsymbol{I}_M$.

The max-min rate optimization problem is then formulated as

$$\max_{\mathbf{p}, \boldsymbol{\phi}} \quad \min_{1 \leq k \leq 2} \log_2(1 + \gamma_k(\mathbf{p}, \boldsymbol{\phi})) \tag{5a}$$

$$\text{s.t.} \quad 0 \preccurlyeq \boldsymbol{\phi} \preccurlyeq 2\pi \tag{5b}$$

$$0 \leq p_k \leq P_k, k = 1, 2, \tag{5c}$$

where P_k is the maximum transmission power of user k.

III. Proposed Algorithm for Max-Min Rate Optimization

Due to the monotonic increasing property of the log function, max-min rate optimization is equivalent to the max-min SINR optimization as

$$\max_{\mathbf{p}, \boldsymbol{\phi}} \quad \min(\gamma_1(\mathbf{p}, \boldsymbol{\phi}), \gamma_2(\mathbf{p}, \boldsymbol{\phi})) \quad \text{s.t. (5b), (5c).} \tag{6}$$

Let us express the SINR expression by applying the Sherman-Morrison formula to $\boldsymbol{B}_1^{-1}(\boldsymbol{p}, \boldsymbol{\phi})$ as

$$\gamma_1(\mathbf{p}, \boldsymbol{\phi}) = \frac{p_1}{\sigma^2}\left[\|\mathbf{h}_1(\boldsymbol{\phi})\|^2 - \frac{|\mathbf{h}_1^H(\boldsymbol{\phi})\mathbf{h}_2(\boldsymbol{\phi})|^2 p_2}{\sigma^2 + \|\mathbf{h}_2(\boldsymbol{\phi})\|^2 p_2}\right] \tag{7}$$

and

$$\gamma_2(\mathbf{p}, \boldsymbol{\phi}) = \frac{p_2}{\sigma^2}\|\mathbf{h}_2(\boldsymbol{\phi})\|^2. \tag{8}$$

From (7) and (8), we have two observations on the power allocation for given $\boldsymbol{\phi}$ as follows:

- $\gamma_1(\mathbf{p}, \boldsymbol{\phi})$ increases with p_1 while it decreases with p_2.
- $\gamma_2(\mathbf{p}, \boldsymbol{\phi})$ is independent of p_1 but increases from zero as p_2 increases.

For better understanding, the SINR behaviors according to the power allocation are shown in Fig. 2 for a given channel realization.

First of all, the power allocation to the stronger user is set to meet $\gamma_1(\mathbf{p}, \boldsymbol{\phi}) = \gamma_2(\mathbf{p}, \boldsymbol{\phi})$, which leads to

$$\tilde{\gamma}_1(1 + \tilde{\gamma}_2) - \tilde{\gamma}_1\tilde{\gamma}_2\rho_{12}^2 = \tilde{\gamma}_2(1 + \tilde{\gamma}_2), \tag{9}$$

where $\tilde{\gamma}_k = p_k\|\mathbf{h}_k(\boldsymbol{\phi})\|^2/\sigma^2$ and $\rho_{12} = \frac{|\mathbf{h}_1^H(\boldsymbol{\phi})\mathbf{h}_2(\boldsymbol{\phi})|}{\|\mathbf{h}_1(\boldsymbol{\phi})\|\|\mathbf{h}_2(\boldsymbol{\phi})\|}$. To meet (9), we have

$$\tilde{\gamma}_2 = \frac{1}{2}\left[\tilde{\gamma}_1(1 - \rho_{12}^2) - 1 + \sqrt{(\tilde{\gamma}_1(1 - \rho_{12}^2) - 1)^2 + 4\tilde{\gamma}_1}\right], \tag{10}$$

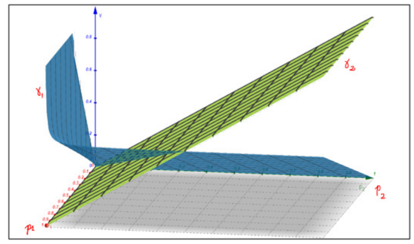

Fig. 2. An example of SINR variations according to power allocation $\boldsymbol{p}$ for given IRS $\boldsymbol{\phi}$.

which has the maximum value when $\tilde{\gamma}_1$ is the maximum with $p_1 = P_1$ while satisfying the condition of $\tilde{\gamma}_2 \leq \frac{P_2}{\sigma^2}\|\mathbf{h}_2(\boldsymbol{\phi})\|^2$. Let us define

$$\tilde{\gamma}_{eq}(\boldsymbol{\phi}) = \tfrac{1}{2}\left[b(\boldsymbol{\phi}) + \sqrt{b(\boldsymbol{\phi})^2 + 4\Gamma_1(\boldsymbol{\phi})}\right], \tag{11}$$

where

$$b(\boldsymbol{\phi}) = \Gamma_1(\boldsymbol{\phi}) - \tfrac{P_1}{\sigma^2}\frac{\left|\mathbf{h}_1^H(\boldsymbol{\phi})\mathbf{h}_2(\boldsymbol{\phi})\right|^2}{\|\mathbf{h}_2(\boldsymbol{\phi})\|^2} - 1 \tag{12}$$

and

$$\Gamma_k(\boldsymbol{\phi}) = \tfrac{P_k}{\sigma^2}\|\mathbf{h}_k(\boldsymbol{\phi})\|^2, \ k = 1, 2. \tag{13}$$

Finally, the problem (6) becomes

$$\begin{aligned} \max_{\boldsymbol{\phi}} \quad & \min(\tilde{\gamma}_{eq}(\boldsymbol{\phi}), \Gamma_2(\boldsymbol{\phi})) \\ \text{s.t.} \quad & 0 \leq \phi_n \leq 2\pi, \forall n. \end{aligned} \tag{14}$$

In solving problem (14), we noitice that it is a nonlinear optimization problem with simple bounded box constraints. If the objective is smooth, the problem can be solved by the L-BFGS-B algorithm which is known to be efficient for a large number of variables. Therefore, we approximate the minimum objective function to a smooth (differentiable) approximation by using

$$\min(x, y) \approx \frac{1}{2}(x + y) - \frac{1}{2}\sqrt{(x - y)^2 + \epsilon}, \tag{15}$$

where $\approx$ becomes the equality as $\epsilon \to 0$.

Finally, we approximate the objective to

$$\begin{aligned} f_{min}(\boldsymbol{\phi}) \simeq & \frac{1}{2}(\Gamma_2(\boldsymbol{\phi}) + \tilde{\gamma}_{eq}(\boldsymbol{\phi})) \\ & - \frac{1}{2}\sqrt{(\Gamma_2(\boldsymbol{\phi}) - \tilde{\gamma}_{eq}(\boldsymbol{\phi}))^2 + \varepsilon}, \end{aligned} \tag{16}$$

of which the gradient is given as follows.

$$\begin{aligned} \boldsymbol{\nabla} f_{min}(\boldsymbol{\phi}) \simeq & \frac{1}{2}\left[1 - \frac{\Gamma_2(\boldsymbol{\phi}) - \tilde{\gamma}_{eq}(\boldsymbol{\phi})}{\sqrt{(\Gamma_2(\boldsymbol{\phi}) - \tilde{\gamma}_{eq}(\boldsymbol{\phi}))^2 + \varepsilon}}\right]\boldsymbol{\nabla}\Gamma_2(\boldsymbol{\phi}) \\ & + \frac{1}{2}\left[1 + \frac{\Gamma_2(\boldsymbol{\phi}) - \tilde{\gamma}_{eq}(\boldsymbol{\phi})}{\sqrt{(\Gamma_2(\boldsymbol{\phi}) - \tilde{\gamma}_{eq}(\boldsymbol{\phi}))^2 + \varepsilon}}\right]\boldsymbol{\nabla}\tilde{\gamma}_{eq}(\boldsymbol{\phi})) \end{aligned} \tag{17}$$

For a given SIC order, we apply the L-BFGS-B algorithm with the gradient obtained numerically or in a closed-form to solve problem (14). For a NOMA pair which has two possible SIC orders, we run the optimization algorithm for each of get SIC order to get the best max-min rate.

IV. Simulation Results

To investigate the performance, we perform the simulation setup in the (x,y,z) coordinates as shown in Fig. 3. The BS is located at (0,0,10) m, the IRS is placed in the 'y-z' axis at (-20,60,10) m, and users are randomly located inside to the 20×20 rectangle centered at (0,60,1.5) m. All the channel links are modeled as Rayleigh fading channels with path loss $\beta d^{-\alpha}$, where the reference path loss is given by $\beta = 10^{-3}$ for all links. The path loss exponent α is given by 2.2, 4, and 2.4 for the BS-IRS, BS-U_k and IRS-U_k links, respectively. The power noise at the BS is set to $\sigma^2 = -100$ dBm. The average performance is obtained over 1000 random CSI samples.

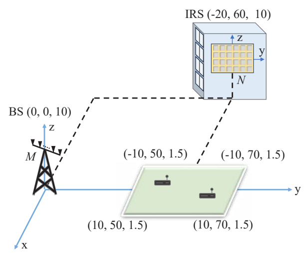

Fig. 3. Simulation setup for an IRS-assisted NOMA pair.

Fig. 4 compares the max-min rate of the two-user NOMA as the number N of IRS elements increases when the number of antennas is given by $M = 1, 2$ and the maximum transmit power for the devices is set to $P_1 = P_2 = 100$ mW. The proposed nonlinear IRS optimization including PA (Prop-IRS/PA) outperforms the nonlinear IRS optimization with maximum PA (Prop-IRS), and the PA without IRS (No-IRS/PA). The performance gain obtained with an IRS gets larger as the number N of IRS elements increases or the number M of antennas increases. The PA gain is more prominent for a small M and N, resulting in a low SNR.

Next, we compare the max-min rate as the maximum transmit power $P = P_k$ varies when $N = 32$ with $M = 1, 2$ in Fig. 5(a) and $M = 1$ with $N = 16, 64$. in Fig. 5(b). Results show that the max-min rate increases as transmit power increases particularly for the optimal PA (Prop-IRS/PA) case.

V. Concluding Remarks

We have considered the IRS-assisted uplink NOMA with two users supported by a multiantenna BS. For the network, the max-min rate was optimized by optimizing IRS reflection, PA, and receive BF. Instead of typical alternating optimization, we proposed a nonlinear optimization with respect to IRS phase shifts by obtaining the optimal receive BF and optimal PA conditions given IRS reflection. The problem with a large-size IRS was solved through the L-BFGS-B algorithm known to be efficient for a large scale problem. Results have revealed that the max-min rate of the IRS-assisted NOMA is improved more prominently as N and M increase. Additionally, optimal PA becomes crucial if M and N are not so large. The proposed optimization based on the L-BFGS-B can be straightforwardly extended for the more general case by jointly optimizing the PA and IRS-phases under the optimal receive BF condition

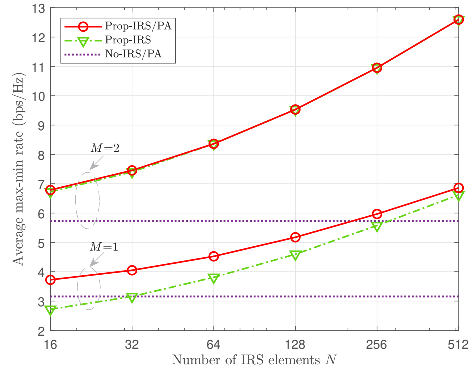

Fig. 4. Average max-min rate as N increase when M = 1,2.

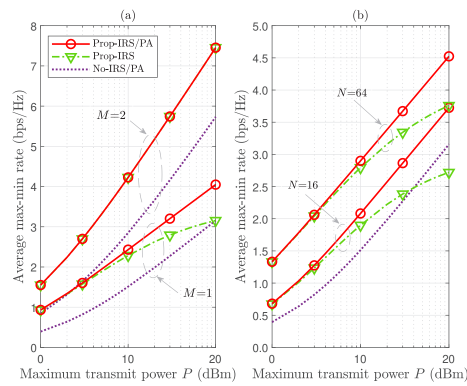

Fig. 5. Average max-min rate as the maximum transmit power P increases when (a) $N = 32$ with $M = 1, 2$ and (b) $M = 1$ with $N = 16, 64$.

since both are box-constrained variables; however when $K > 2$ obtaining the optimal PA conditions and SIC order is not an easy task thus requiring $N + K$ variables in general.

References

[1] Q. Wu, S. Zhang, B. Zheng, C. You, and R. Zhang, "Intelligent Reflecting Surface Aided Wireless Communication: A tutorial," *IEEE Trans. on Comm.*, vol. 69, no. 5, pp. 3313-3351, May 2021.

[2] G. Yang, X. Xu, Y. Liang, and M. Renzo, "Reconfigurable intelligent surface-assisted non-orthogonal multiple access," *IEEE Trans. Wirel. Commun.,* vol. 20, no. 5, pp. 3137–3151, May 2021.

[3] X. Xie, F. Fang, and Z. Ding, "Joint optimization of beamforming, phase-shifting and power allocation in a multi-cluster IRS-NOMA network," *IEEE Trans. Veh. Technol.,* vol. 70, pp. 7705–7717, Aug. 2021.

[4] M. Zeng, X. Li, G. Li, W. Hao, and O. A. Dobre, "Sum rate maximization for IRS-assisted uplink NOMA," *IEEE Commun. Lett.*, vol. 25, no. 1, pp. 234–238, Jan. 2021.

[5] J. Choi, L. Cantos, J. Choi, and Y.H. Kim, "Sum rate optimization of IRS-aided uplink muliantenna NOMA with practical reflection," *Sensors*, vol. 22, no. 4449, June 2022.

[6] L. Cantos, J.H. Choi, and L. Cantos, "Fairness optimization for IRS-assisted uplink with two-user NOMA," *Proc. KICS Winter Confer.,* pp. 1, Feb. 2022.

[7] L.Cantos, M. Awais, and Y.H. Kim, "Max-min rate optimization for uplink IRS-NOMA with receive beamforming," *IEEE Wireless Commun. Lett.*, vol 11, no 12, pp. 2512–2516, Dec 2022.

[8] J. Nocedal, S. Wright, "Numerical optimization", 2nd ed. Springer, New York, NY, 2006.

[9] C. Lee and Y. H. Kim, "Receive beamforming and resource allocation for wireless powered non-orthogonal multiple access," *IEEE Trans. Veh. Technol.*, vol. 69, no. 4, pp. 4563–4568, Apr. 2020.