# Computational Complexity Analysis of Beamspace Transformation for Dual Array Antennas


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***Abstract*— 5G mobile communication using a high carrier frequency can operate a massive array antenna. A large-scale antenna element is a major factor that increases the hardware and software complexity of an array antenna. In such a massive array antenna, beamspace conversion is a very efficient method to reduce the high complexity of element space. This paper presents a mathematical model that analyzes the total number of addition/subtraction and multiplication/division of beamspace transformation formulas for a dual array antenna among various massive array antennas and provides results of computational complexity analysis.



***Keywords—dual arrya anteena; dual frame array antenna; dual concentric circular arrya; beamspace transformation; computational complexity analysis***


## I. Introduction

The size of the antenna element is in inverse proportion to the carrier frequency used by the system. That is, next-generation mobile networks including 5G communication using high carrier frequencies can utilize antenna elements having a very small size. As more antenna elements can be utilized in the same area, next-generation mobile networks can apply massive array antennas. Many antenna elements increase hardware and software complexity, but software complexity can be effectively reduced through beamspace conversion [1],[2]. There are various shapes of massive antennas such as square, circular, octagonal, etc.[3]-[5], but this paper aims to analyze the beamspace conversion complexity of a dual array antenna with high spatial efficiency. For the dual array antenna considered in the paper, a Dual Frame Array (DFA) antenna and a Dual Concentric Circular Array (DCCA) antenna were considered, and a mathematical model was presented for complexity analysis.

## II. Anetenna Model

Compared to Filled Array (FA : commonly used Uniform Rectangular Array (URA), Uniform Circular Array (UCA), Uniform Hexagonal Array (UHA), etc.), which use all the empty space inside the array geometry, Rim Array (RA) are continuously researched due to their advantages of using fewer sensors, similar performance to FA, and lower cost [6],[7]. If the center of a rim array is arranged equally many times, it has the form of a Concentric Rim Array (CRA).

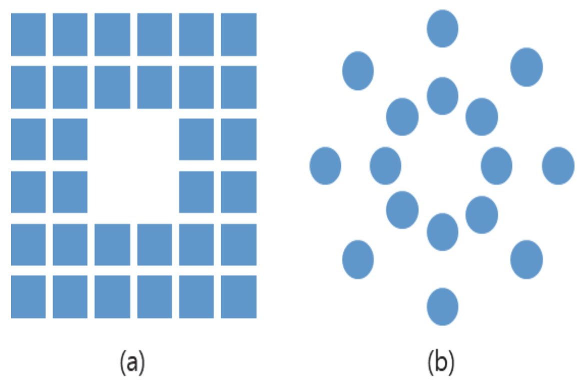


Fig. 1. Dual Array Antenna: (a) Dual Frame Array Antenna, (b) Dual Concentric Circular Array Antenna

The dual array antenna model with the CRA shape considered in the paper is shown in Fig. 1. The first DFA antenna considered is a form in which Boundary Array (BA) antennas that utilize the edge elements of URA antenna elements are composed of two layers. DCCA is a form in which UCA having different radii is arranged in a concentric circle structure.

## III. Beamspace model for computational complexity analysis

In this chapter, we present a beamspace transform matrix and computational complexity mathematical model for analyzing the beamspace transform complexity of DFA and DCCA antennas.

### *A. Beamspace Model of DFA antenna*

A typical URA antenna defines a beamspace transformation matrix through a kronecker product of Discrete Fourier Transform (DFT) matrices of x-axis and y-axis. However, as shown in Fig. 1(a), since the DFA antenna does not have a central element, the kronecker product of (1) and (2) must be independently performed after dividing the DFA antenna into 4 subarrays to generate a beamspace transformation matrix [8]. In (1) and (2), $M_x$ is the number of antenna elements arranged on the x-axis, $M_y$ is the number of antenna elements arranged on the y-axis, $\mathbf{b}_x$ is the DFT matrix for the x-axis, and $\mathbf{b}_y$ is the DFT matrix for the y-axis.

$$\mathbf{BD}_{11,33} = \frac{1}{\sqrt{2M_x}} \mathbf{b}_y \otimes \mathbf{b}_x , \tag{1}$$

$$\mathbf{BD}_{22,44} = \frac{1}{\sqrt{2M_y}} \mathbf{b}_x \otimes \mathbf{b}_y , \tag{2}$$

The beamspace transformation matrix $\left(\mathbf{BT}_{DFA} \in \mathbf{R}^{BD \times M}\right)$ for the DFA antenna is defined as

$$\mathbf{BT}_{DFA} = \begin{bmatrix} \mathbf{BD}_{11} & \mathbf{0}_{12} & \mathbf{0}_{13} & \mathbf{0}_{14} \\ \mathbf{0}_{21} & \mathbf{BD}_{22} & \mathbf{0}_{23} & \mathbf{0}_{24} \\ \mathbf{0}_{31} & \mathbf{0}_{32} & \mathbf{BD}_{33} & \mathbf{0}_{34} \\ \mathbf{0}_{41} & \mathbf{0}_{42} & \mathbf{0}_{43} & \mathbf{BD}_{44} \end{bmatrix}, \tag{3}$$

*B. Beamspace Model of DCCA antenna*

Since the DCCA antenna is a combination of UCA antennas with different number of antennas, the beamspace transformation matrix of each UCA antenna must be defined as [9]

$$\mathbf{BU}_{11} = \frac{1}{U_1} \mathbf{W}^H \mathbf{C} \mathbf{P}^H , \tag{4}$$

$$\mathbf{BU}_{22} = \frac{1}{U_2} \mathbf{W}^H \mathbf{C} \mathbf{P}^H , \tag{5}$$

In (4) and (5), $U_n \left(n = 1, 2\right)$ is the number of antenna elements in each circular array, $\mathbf{W}$ is a weight matrix, $\mathbf{C}$ is a scale matrix, and $\mathbf{P}$ is a phase mode matrix.

The beamspace transformation matrix $\left(\mathbf{BT}_{DCCA} \in \mathbf{R}^{BU \times U}\right)$ for the DCCA antenna is defined as

$$\mathbf{BT}_{DCCA} = \begin{bmatrix} \mathbf{BU}_{11} & \mathbf{0}_{12} \\ \mathbf{0}_{21} & \mathbf{BU}_{22} \end{bmatrix}, \tag{6}$$

*C. Computational complexity of beamspace transformation*

The computational complexity for defining the beamspace matrix for each array antenna is summarized in Table 1. The DFA antenna uses only the kronecker product of each sub-array when defining the beamspace matrix, so complexity for addition/subtraction does not occur. In Table 1, $x_b$ and $y_b$ represent the order of the DFT matrix, $\mu_n = k_0 r_n \left(n = 1, 2\right)$ represents the highest order term of the excitation mode, $k_0$ is the wave number, $r_n$ is the radius of the nth circular array.

*D. Computational complexity covariance matrix*

The covariance matrix applied to various signal processing techniques is defined as [10]

$$\begin{aligned} \mathbf{R}(k) &= \frac{1}{k} \sum_{i=1}^{k} \mathbf{x}(i) \mathbf{x}^H(i) \\ &= \frac{k-1}{k} \mathbf{R}(k-1) + \frac{1}{k} \mathbf{x}(k) \mathbf{x}^H(k) \end{aligned}, \tag{7}$$

TABLE I. THE COMPUTATIONAL COMPLEXITY OF BEAMSPACE TRANSFORMATION MATRICES APPLIED TO DFA AND DCCA ANTENNAS

| Index | Computational complexity |
|---|---|
| $C_{DFA(+/-)}$ | 0 |
| $C_{DFA(\times/\div)}$ | $\left(x_b y_b M_x M_y\right)_{sub1} + \left(x_b y_b M_x M_y\right)_{sub2} + \left(x_b y_b M_x M_y\right)_{sub3} + \left(x_b y_b M_x M_y\right)_{sub4}$ |
| $C_{DCCA(+/-)}$ | $\left(4\mu_1^2 + 2\mu_1\right)\left(2\mu_1 + U_1\right) + \left(4\mu_2^2 + 2\mu_2\right)\left(2\mu_2 + U_2\right)$ |
| $C_{DCCA(\times/\div)}$ | $\left(2\mu_1 + 1\right)^2 \left(2\mu_1 + U_1 + 1\right) + \left(2\mu_2 + 1\right)^2 \left(2\mu_2 + U_2 + 1\right)$ |

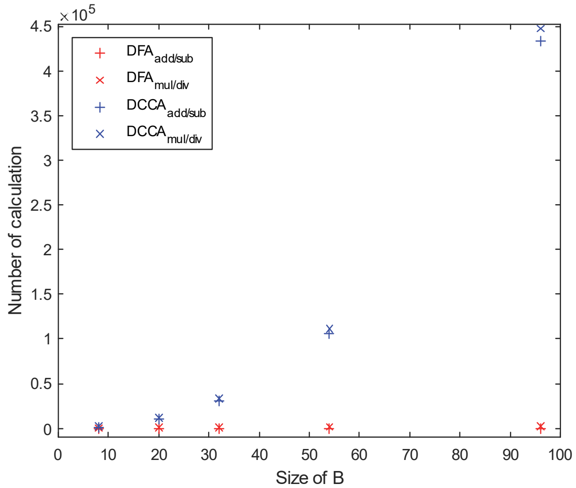


Fig. 2. Comparison of beamspace transform complexity of DFA and DCCA antennas according to beamspace transform matrix size

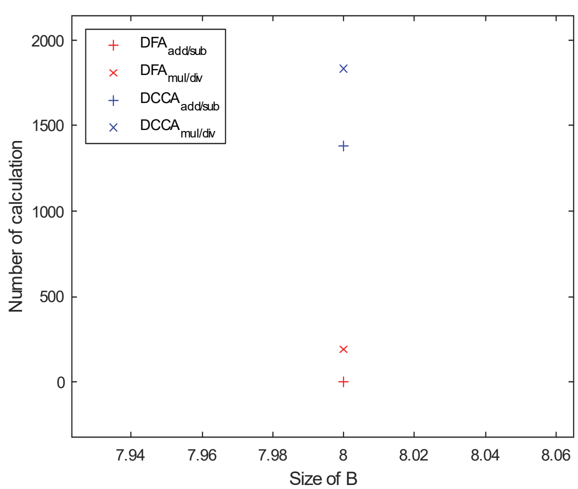


Fig. 3. Extenstion of Fig. 2 when the beamspace transformation matrix size is 8

$$\begin{aligned}\mathbf{R}_B(k) &= \frac{1}{k}\sum_{i=1}^{k}\mathbf{x}_B(i)\mathbf{x}_B^H(i) \\ &= \frac{k-1}{k}\mathbf{R}_B(k-1) + \frac{1}{k}\mathbf{x}_B(k)\mathbf{x}_B^H(k)\end{aligned}, \quad (8)$$

where $\mathbf{R}(k)$ is element-space covariance matrix, $\mathbf{R}_B(k)$ is beamspace covariance matrix, $\mathbf{x}(k)$ and $\mathbf{x}_B(k)$ are element-space and beamspace received signal vectors having sizes $M \times 1$ and $B \times 1$ for the kth sample index. For each sample index $k$, the multiplication/division and addition/subtraction

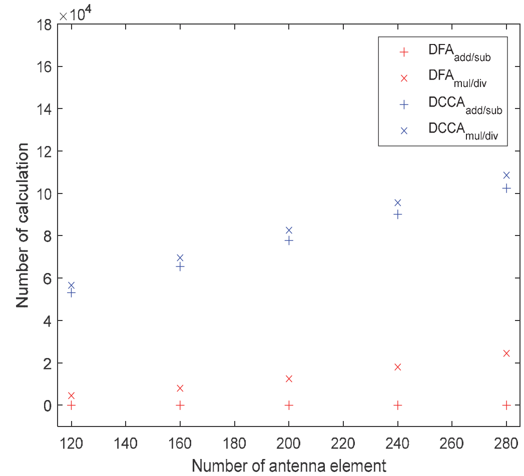


Fig. 4. Comparison of beamspace transform complexity of DFA and DCCA antennas according to number of antenna elements

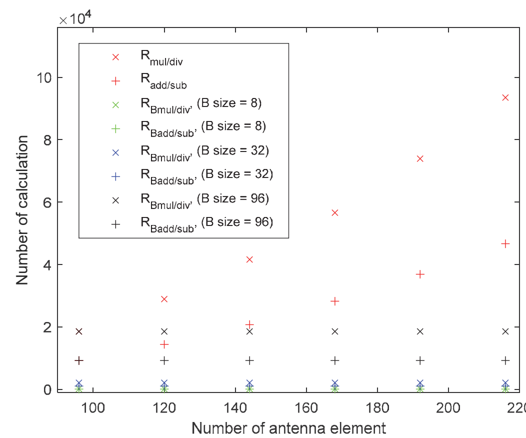


Fig. 5. Comparison of computational complexity of element-space and beamspace covariance matrices according to the number of antennas

complexities of (7) and (8) are given by $2M^2 + M$, $M^2$ and $2B^2 + B$, $B^2$ respectively.

## IV. Computer Simulation

For the simulation of Figure 2, the number of antenna elements was assumed to be 96 considering the number of the DFA antenna has 44 antenna elements in the inner array and 52 antenna elements in the outer array, while the DCCA antenna has 45 antenna elements in the inner array and 51 antenna elements in the outer array. To compare the size of the common beamspace transformation matrix of each array antenna, the sizes of the beamspace transformation matrix $(BD, BU)$ were set to 8, 20, 32, 54, and 96.

Fig. 2 shows the simulation results performed according to the previously set simulation conditions. Fig. 3 is an extended view of Fig. 2 when the size of the beamspace matrix is 8. Fig.4 shows the computational complexity of the DFA and DCCA antennas while keeping the beamspace size fixed at 16 and increasing the number of antenna elements by 40 from 120 to 280. Fig. 5 compares the computational complexity of the element-space and beamspace covariance matrix according to the number of antenna elements. For the simulation, the number of antenna elements was increased from 96 to 216 in increments of 24, and the beamspace size was set to 8, 32, and 96. From the result of the simulation in Fig.2 and Fig.3, when the size of the beamspace matrix is very small compared to the antenna element, the difference in computational complexity between the DFA antenna and the DCCA antenna is not large. On the other hand, when the size of the beamspace is the same as that of the antenna element, the beamspace transformation complexity of the DFA antenna does not change significantly, but the beamspace transformation complexity of the DCCA antenna becomes very large. From the results in Fig. 4 that as the number of antennas is progressively increased, the complexity also increases. However, despite the small size of the beamspace matrix, the DCCA antenna has a higher complexity than the DFA antenna throughout the range assumed in the simulation. As can be seen in the Fig.5, as the number of antennas increases, the computational complexity of the element space covariance matrix gradually increases. However, the computational complexity of the beam space covariance matrix is affected by the initially set size of the beam space matrix, and has a constant complexity regardless of the number of antenna elements.

The simulation results show that for all sizes of beamspace conversion matrices assumed in the paper, the DFA antenna has relatively less complexity than the DCCA antenna. However, the DFA antenna is limited in the size of the beamspace conversion matrix because the beamspace conversion matrix is generated through the kronecker product, but the DCCA antenna generates the beamspace conversion matrix in stages by adjusting the highest order term of the phase mode, so the matrix size is relatively less limited.

In actual use, there is no case where the size of the beamspace is the same as that of the antenna element, and in order to reduce complexity, the size of the beamspace takes a very small value compared to the number of antenna elements. That is, in massive array antennas, it is common for $B \ll M$. In addition, since the size of the beamspace transformation matrix is directly related to the number of signals that can be estimated when combined with the Angle of Arrival(AOA) estimation algorithm later, a beam space transformation matrix of an appropriate size must be selected. As such, each antenna array has advantages and disadvantages, so it is necessary to properly select an antenna suitable for the system situation.

## V. Conclustion

Beamspace conversion is a technique that can efficiently alleviate a problem of high computational complexity that may occur in element space due to the large number of antenna elements. This paper analyzed the complexity of the beam space transformation matrix of the CRA type DFA antenna and DCCA antenna among various types of massive array antennas. From the simulation results, it is determined that the DFA antenna using only kronecker product can effectively reduce the complexity because the computational complexity is lower than that of the DCCA antenna. However, since the constraint on the size of the beam space transformation matrix is larger than that of the DCCA antenna, it is necessary to select an appropriate antenna by reflecting the strengths and weaknesses of the antenna and the physical situation of the system.

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