Service-Based Optimal Group Resource Allocation Strategy

Chaeyeon Cha
Graduate Program in Smart Factory
Dept. Electronic and Electrical Engineering
Ewha Womans University
Seoul, Republic of Korea
chaeyeon.cha@ewhain.net

Hyunggon Park
Graduate Program in Smart Factory
Dept. Electronic and Electrical Engineering
Ewha Womans University
Seoul, Republic of Korea
hyunggon.park@ewha.ac.kr

***Abstract*—With the increasing development of the Internet of Things (IoT), there is a growing need for efficient resource allocation to the many devices in the IoT system. This need is particularly acute in the case of time-sensitive applications, where fast resource allocation is essential. However, the resource allocation based on the Nash bargaining solution (NBS) necessitates exponentially increasing computational complexity for user dynamic systems. We propose an approach that makes groups based on the requested services and predicts a disagreement point whenever the group size changes, with the aim of reducing the computational complexity to find the NBS. Through simulation results, we demonstrate that our proposed approach outperforms the existing method in terms of execution time to find the NBS, maintaining the optimality in finding NBS.**

***Index Terms*—Internet of Things, Nash bargaining solution, resource management.**

I. Introduction

As the Internet of Things (IoT) technology has been developed, various devices are being interconnected, exchanging real-time information and interacting with each other, leading to a significant increase in the number of dynamic systems [1], [2]. Due to the scalability support of IoT systems and the scarce resources, multiple users optimally should share limited resources considering dynamic interactions. Hence, appropriate resource management and scheduling strategies for efficient resource sharing among users are essential [3].

The Nash Bargaining Solution (NBS) can offer a fair and efficient approach to resource management problems. It has been adopted as a resource allocation strategy in various fields for multi-user resource allocation including in IoT systems. For example, NBS is utilized for power distribution in IoT nodes [4], incentivizing cooperative provision of computing in cloud-edge IoT frameworks [5], and enabling long-term proportional fairness-driven fifth-generation edge healthcare [6]. Furthermore, the NBS is used to optimize overall system performance [7] as it maximizes the Nash product (NP), the multiplication of achievable utilities.

The utility, representing the subjective value that each user assigns to particular resources, is generally mapped to a concave function due to diminishing marginal utility [8]. Specifically, the multiplicatively concave function is defined by replacing the weighted arithmetic means with the weighted geometric means [9]. It is known that the inequalities based on multiplicative convexity are better than the direct application of the usual inequalities of convexity [10]. Because this property of the multiplicatively concave function can yield monotonicity of certain sequences, the multiplicatively concave function has been used in a variety of fields to analyze problems involving uncertainty, risk premiums, certainty equivalents, and relative risk aversion [11].

In dynamic systems, resource allocation based on NBS faces a significant challenge in terms of computational complexity, since the complexity increases exponentially as the number of users changes. This becomes even more challenging if the number of users is changing and the NBS should be computed repeatedly. Intra NBS [12] addresses this issue by using the axiom of independence of linear transformations in NBS. It can efficiently reduce the computational complexity of finding the NBS by setting the disagreement point using the relation between the adjacent NBS. However, Intra NBS has a limitation that it can only be utilized when the number of users does not change, so it is unable to be used in IoT systems where the number of users varies. To address this limitation of Intra NBS in IoT systems, we propose a method of creating groups based on the services that users request while maintaining the optimality of the resource allocation solution, and we propose to predict the disagreement point when the number of users in group changes.

The remainder of the paper is organized as follows. In Section II, we describe the problem considered in this paper. Then, the proposed algorithm is discussed with the convexity and the process in Section III. In Section IV, we show the performance comparison of the proposed algorithm compared to Intra NBS. The conclusions are drawn in Section V.

II. Problem Setup

We consider a dynamic system in which a local server manages resources in a designated coverage. At time slot t, there are $n(t)$ users that share time-varying but limited resources $R(t)$. Every user requests a service to the local server, and the local server makes distinct groups G_i $(i = 1, \dots, g)$ for the users who request the same service. The number of groups cannot exceed the number of users, i.e., $1 \leq g \leq n(t)$.

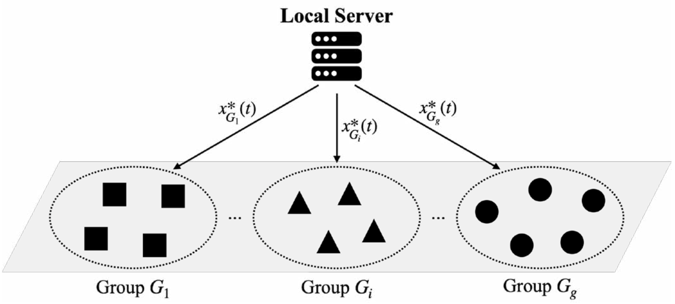


Fig. 1: The considered dynamic system with service-based groups

An illustrative example of the considered dynamic system is shown in Fig. 1.

The number of users belonging to group G_i is denoted by $m_j(t)$, i.e., $\sum_{i=1}^{g} m_j(t) = n(t)$. The amount of resources allocated to user j in a group is denoted by $x_j(t)$. Since all users in a group request the same service, every user in a group has the same utility function, $u_j(x_j(t))$, that can be achieved by the allocated resource $x_i(t)$, i.e.,

$$u_j(x_j(t)) = u_k(x_k(t))$$

for all users in a group where $j \neq k$.

In this paper, we define the group utility function, $u_{G_i}(x_{G_i}(t))$, as the product of the utilities from all users in the group, i.e.,

$$u_{G_i}(x_{G_i}(t)) = \prod_{j=1}^{m_i(t)} u_j\left(x_j(t)\right),$$

where the amount of resources allocated to G_i at time t is denoted by $x_{G_i}(t)$. All users in G_i share the resources $x_{G_i}(t)$ equally, i.e., $x_j(t) = \frac{x_{G_i}(t)}{m_i(t)}$ $(j = 1, \ldots, m_i(t))$. Then, the feasible utility set $\mathbf{S}_G(t)$, defined as a set of the feasible utility pairs for all groups with potential resource allocations, is correspondingly expressed as

$$\mathbf{S}_G(t) = \left\{ \mathbf{U}_G(\mathbf{x}_G(t)) \,\middle|\, \sum_{i=1}^{g} x_{G_i}(t) \leq R(t) \right\}, \tag{1}$$

where $\mathbf{U}_G(\mathbf{x}_G(t)) = (u_{G_1}(x_{G_1}(t)), \ldots, u_{G_g}(x_{G_g}(t))) \in \mathbb{R}_+^g$. Then, the product of group utilities is defined by

$$G_G(\mathbf{x}_G(t)), \mathbf{d}_G = \prod_{i=1}^{g} u_{G_i}(x_{G_i}(t) - d_{G_i}),$$

where $\mathbf{x}_G(t) = (x_{G_1}(t), \ldots, x_{G_g}(t)) \in \mathbb{R}_+^g$ and $\mathbf{d}_G = (d_{G_1}, \ldots, d_{G_g}) \in \mathbb{R}_+^g$. For simplicity, we assume that the disagreement point is $\mathbf{d}_G = \mathbf{0}$ in this paper. The NBS determined by the maximizer of the NP, i.e.,

$$\mathbf{X}_G^*(t) = \underset{\mathbf{1}^T \mathbf{x}_G(t) \leq R(t)}{\arg\max} \; G_G(\mathbf{x}_G(t), \mathbf{d}_G),$$

where $\mathbf{X}_G^*(t) = (u_{G_1}(x_{G_1}^*(t)), \ldots, u_{G_g}(x_{G_g}^*(t)))$.

In this paper, we focus on designing an algorithm for the optimal resource allocation $\mathbf{x}_G^*(t)$ with lower computational complexity if the group utility functions are multiplicatively concave and increasing concave. For this, we reduce the search space of the feasible utility function by predicting the consecutive disagreement points while maintaining the optimality of the resource allocation solution.

III. Proposed Solution

The key idea of the proposed algorithm is the prediction of the disagreement point $\mathbf{d}_G$ of current NBS $\mathbf{X}_G^*(t)$ from the past NBS $\mathbf{X}_G^*(t-1)$, which can reduce the search space in $\mathbf{S}_G(t)$. The dimension of the feasible utility set at time slot t is assumed to be the same as the dimension of it at time slot $t-1$, i.e., $dim(\mathbf{S}_G(t)) = dim(\mathbf{S}_G(t-1))$.

A. Convexity of the feasible utility set

In order to guarantee to find the NBS, the feasible utility set in (1) needs to be convex [13]. As shown in Lemma 1, the feasible utility set of $\mathbf{S}_G(t)$ is convex. Note that we drop the notation of time slot t in the proof of this section for a succinct description.

Lemma 1. *The feasible utility set $\mathbf{S}_G$ is convex.*

Proof. It is known that if there exists an interval I which is a subinterval of $(0, \infty)$, a differentiable function $f : I \to (0, \infty)$ is multiplicatively concave if and only if

$$f(x^\theta y^{1-\theta}) \geq \{f(x)\}^\theta \{f(y)\}^{(1-\theta)}$$

for all $x,\ y \in I$ and $\theta \in [0, 1]$. To show the convexity of the set $\mathbf{S}_G$, we should show that for any $\theta \in [0, 1]$,

$$\begin{aligned} &\mathbf{X}_G = (X_{G_1}, \ldots, X_{G_g}) = (u_{G_1}(x_{G_1}), \ldots, u_{G_g}(x_{G_g})) \in \mathbf{S}_G \\ &\mathbf{Y}_G = (Y_{G_1}, \ldots, Y_{G_g}) = (u_{G_1}(y_{G_1}), \ldots, u_{G_g}(y_{G_g})) \in \mathbf{S}_G \\ &\quad \Rightarrow \theta \mathbf{X}_G + (1-\theta)\mathbf{Y}_G \in \mathbf{S}_G \end{aligned}.$$

The ith element of $\theta \mathbf{X}_G + (1-\theta)\mathbf{Y}_G$ is $\theta X_{G_i} + (1-\theta) Y_{G_i}$. Since group utility function u_{G_i} is multiplicatively concave and increasing concave, its inverse function $u_{G_i}^{-1}$ is increasing convex [14]. Since $u_{G_i}^{-1}$ is convex,

$$\begin{aligned} &u_{G_i}^{-1}(\theta X_{G_i} + (1-\theta) Y_{G_i}) \\ &\qquad \leq \theta u_{G_i}^{-1}(X_{G_i}) + (1-\theta) u_{G_i}^{-1}(Y_{G_i}) \\ &\qquad = \theta u_{G_i}^{-1}(u_{G_i}(x_{G_i})) \\ &\qquad\qquad + (1-\theta) u_{G_i}^{-1}(u_{G_i}(y_{G_i})) \\ &\qquad = \theta x_{G_i} + (1-\theta) y_{G_i} \end{aligned}$$

Hence,

$$\begin{aligned} \sum_{i=1}^{g} u_{G_i}^{-1}(\theta X_{G_i} + (1-\theta) Y_{G_i}) &\leq \sum_{i=1}^{g} \theta x_{G_i} + (1-\theta) y_{G_i} \\ &= \theta \sum_{i=1}^{g} x_{G_i} + (1-\theta) \sum_{i=1}^{g} y_{G_i} \\ &\leq \theta R + (1-\theta) R = R \end{aligned}$$

Therefore, $\mathbf{S}_G$ is convex. $\square$

B. *Predicted disagreement point*

We use the axiom of *independence of linear transformations* for NBS [13] to predict the disagreement point at time slot t. It is shown that the changes in the feasible utility sets from $\mathbf{S}_G(t-1)$ and $\mathbf{S}_G(t)$ can be captured by a linear transformation matrix $\mathbf{T}(t)$ [12], expressed as

$$\mathbf{T}(t) = \begin{bmatrix} \frac{u_{G_1}(R(t))}{u_{G_1}(R(t-1))} & 0 & \cdots & \cdots & 0 \\ 0 & \frac{u_{G_2}(R(t))}{u_{G_2}(R(t-1))} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \frac{u_{G_g}(R(t))}{u_{G_g}(R(t-1))} \end{bmatrix}.$$

Hence, the predicted disagreement point $\hat{\mathbf{d}}_G(t)$ at time slot t can be expressed as

$$\hat{\mathbf{d}}_G(t) = \mathbf{T}(t) \cdot \mathbf{X}^*_G(t-1). \tag{2}$$

However, the usage of the predicted disagreement point in (2) is limited to cases where the group utility functions at time slot $t-1$ and t are consistent, meaning that the number of users in each group remains constant between these two-time slots. In order to predict the disagreement point when the number of users changes, we compute the NBS $\mathbf{X}^*(t-1)$ at time slot $t-1$ with direction vector-based algorithm [15]. This approach enables us to predict the disagreement point $\hat{\mathbf{d}}_G(t)$ even when the number of users in each group fluctuates.

IV. Experiment Results

A. *Simulation Setup*

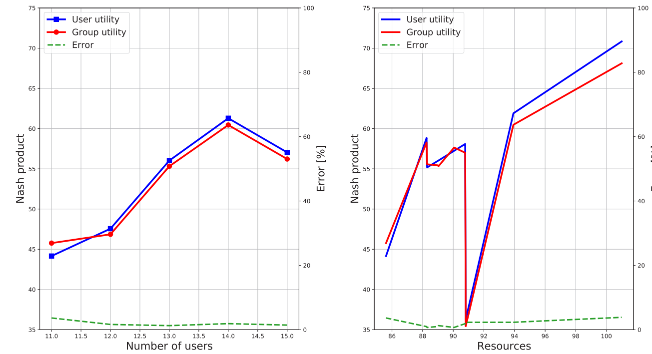
For the performance evaluation of the proposed algorithm, we consider a shared network with multiple users sharing time-varying resources. For the dynamics of the number of users, we use a simple state transition probability matrix, $\mathbf{P} = [p_{ij}]$ for $1 \le i, j \le 2$, where p_{ij} represents the probability of a transition from state i to state j. The states 1 and 2 represent stationary and mobile populations, respectively. Since the user dynamics may be immutable in sufficiently small time slots, $\mathbf{P}$ can be characterized by

$$\mathbf{P} = \begin{bmatrix} 0.9 & 0.1 \\ 0.8 & 0.2 \end{bmatrix},$$

The number of groups is 3 ($g = 3$) and the group utility function is represented by

$$u_{G_i}(x_{G_i}(t)) = \left\{x^{1-a_i}\right\}^{m_i(t)}$$

where a_i $(1 - \frac{1}{m_i(t)} < a_i < 1)$ is the degrees of risk aversion.



(a) User dynamics (b) Resource dynamics

Fig. 2: NP and error obtained by using user utilities and group utilities with user dynamics and resource dynamics over t

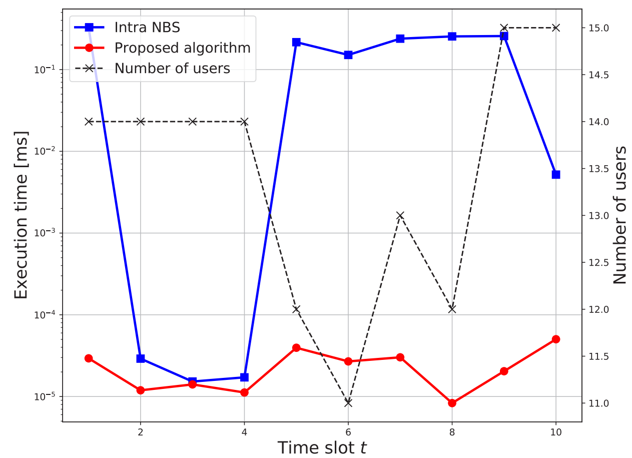


Fig. 3: Execution time required to compute the NBS as the number of users changes over time slot t

B. *Simulation Results*

To analyze the computational accuracy, we compare the NPs obtained by using individual user utilities and group utilities. Note that an algorithm that can achieve a higher NP can be considered as a better algorithm since the NBS maximizes the NP. Fig. 2 shows the NPs and the errors obtained by the individual user utilities and group utilities as the number of users changes or resources change over time. It is observed that there are only small errors (at most 5%) between the NP obtained by user utility and group utility. Hence, the proposed algorithm can maintain the optimality of NBS.

For performance comparison, we measure the actual execution time of Intra NBS [12] and the proposed algorithm to find the NBS. The simulation results in Fig. 3 show the performance evaluation of the NBS algorithms over time-varying resources with user dynamics. It is clearly shown that the proposed algorithm requires a shorter time to find NBS than Intra NBS. Unlike the Intra NBS, which is unable to be adopted in user dynamics, the proposed algorithm predicts the disagreement point of the feasible utility set when the number of group members changes.

V. Conclusion

In this paper, we propose a service-based group resource allocation algorithm, which can find the NBS with lower complexity in the system with user and resource dynamics. The key idea of the proposed algorithm is to predict the disagreement point in the next feasible utility sets of dynamic service-based groups, reducing the search space to find the NBS. We analytically demonstrate that the feasible utility set for group utility is convex. Moreover, the designed group utility function can still guarantee the optimality in finding NBS.

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