# Interference-Limited Multiuser Photon-counting Channel with Incomplete Information: A Bayesian Game Approach for Optimum Transmission

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***Abstract*—In this paper, we consider a multiuser photon-counting Poisson channel and focus on a game-theoretic approach that permits each user with conflicting objectives to decide transmission strategies. We consider a block fading multiuser channel and characterize the received signal as a continuous-time photon-counting receiver. In particular, to account for the incomplete information due to the randomness in the channel, we model the problem as a Bayesian game and analyze the interaction of users with conflicting objectives. With peak and average power constraints imposed and assuming that each user is only aware of the probability distribution of the interfering channel gains, different transmit strategies corresponding to channel randomness are illustrated. It is demonstrated that the transmission strategy of any user is a function of the number of users present in the network. Moreover, it is shown that with appropriate selection of the transmit strategy, all users can improve their quality of service by minimizing the level of interference.**

***Index Terms*—Bayesian, Game-theoretic, interference-limited channel, multiuser, photon-counting receiver, Poisson channel.**

## I. Introduction

In the past few years, free-space optical wireless communication has experienced strong growth, as it offers a license-free and cost-effective solution for high data rate communication systems, and recently has been applied for indoor and outdoor wireless communication applications with wavelength ranging from visible light to infrared to ultraviolet [1–4]

For some optical wireless applications, it is widely known that due to the strong atmospheric fading and path loss, the received signal strength becomes weak and cannot be detected by conventional wide-band continuous waveform receivers. Instead, a photon-counting detector must be employed for systems with weak received signal intensity. In such scenarios, it is common to model the received optical signal as a Poisson process. The Poisson channel is commonly used to model direct detection and low-intensity optical channels [5].

Understanding both propagation channel model and its characterization is critical in accurately analyzing the performance of optical wireless communication systems. Generally, a practical channel for low-power free-space optical wireless communication is modeled as a photon-counting Poisson channel [6]. In addition, in optical scattering communications, it is difficult to detect the received optical signal using a continuous waveform detector due to large atmospheric attenuation. Therefore, a photon-counting receiver is viewed as a more appropriate detector where the number of discrete photoelectrons obeys Poisson distribution [7].

Recently, multiuser optical wireless communication systems have received much attention. A few researchers proposed a precoding algorithm to minimize the interference from users. However, the focus was placed only on maximizing the sum data rate, with the fairness of achievable data rate among users ignored [8]. In addition, as the number of users increases, the interference increases. This, in turn, leads to poor quality of service, i.e., a low signal-to-interference-plus-noise ratio (SINR), and even sometimes results in a failure of service. To tackle this impairment, we recently proposed a capacity maximization technique based on a game-theoretic approach for multiuser interference-limited Poisson channel

[9]. In particular, a Nash bargaining solution from a cooperative game-theoretic perspective was presented with a solution corresponding to the point on the Pareto boundary obtained. Game theory is an analytical tool that offers a suite to analyze the interaction between rational decision-making players (users) [10].

In this work, it is assumed that each user has no knowledge of the interfering channels gain, and is only aware of the probability distribution of the interfering channels. We aim to improve the quality of transmission by reducing multiuser interference in an optical communication system with self-interested rational users having conflicting objectives. With incomplete information about the channel gain, we formulate the problem as a Bayesian game. In particular, a game-theoretic approach is investigated to maximize the data rate while avoiding the collision by strategically choosing an appropriate transmission action.

### A. Contributions

The main contributions of this paper are listed below.

- We consider a multiuser communication system in an interference-limited Poisson channel. A block fading channel is considered and we characterize the received signal as a continuous-time photon-counting receiver. An altruistic resource allocation problem with transmit power constraints imposed, is interpreted as a Bayesian game. In particular, to account for the incomplete information due to the randomness in the channel, we model the problem as a Bayesian game and analyze the interaction of users with conflicting objectives.
- Moreover, we provide the results to analyze the region enclosing the achievable rate in a multiuser power-limited Poisson channel to analyze the problem of capacity maximization.

The rest of the paper is organized as follows. System model and signal characterization are illustrated in Section II. Section III presents the Bayesian game formulation and discusses different transmit strategies. Results are depicted in Section IV. Finally, conclusions are drawn in Section V.

## II. Signal Characterization

As illustrated in Fig. 1, we consider a multiuser interference-limited Poisson channel with $N_u$ number of users. Let $\{x_i(t), t \geq 0\}$ represent $\mathbb{R}_0^+-$valued input signal to the $i$th transmitter, where $i \in \{1,2,\cdots,N_u\}$. We impose the restriction on $x_i(t)$ such that the input to the $i$th transmitter is non-negative and less than a predetermined threshold $P_{t,i}$, where $P_{t,i}$ can be thought of as peak transmit power. Mathematically, $x_i(t)$ satisfies the following constraints. $0 \leq x_i(t) \leq P_{t,i}$ and $\frac{1}{T_x}\int_0^{T_x} E[x_i(t)]dt \leq \delta P_{t,i}, \forall t, 0 \leq t \leq T_x$ and for $i \in \{1,2,\cdots,N_u\}$. $T_x$ denotes the pulse duration of the transmitted non-negative code word waveform. $\delta$ is a ratio of average-to-peak power. Let $\{y_i(t), t \geq 0\}$ be the signal received at the $i$th receiver. $y_i(t)$ is a $\mathbb{Z}_0^+-$valued non-negative left-continuous non-decreasing doubly stochastic Poisson process with the instantaneous rate given by

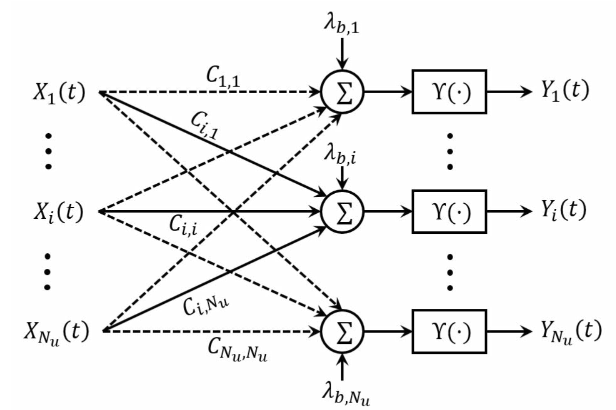


Fig. 1. System illustration: Interference-limited multiuser photon-counting Poisson channel.

$$\mathscr{A}_i = c_{i,i}(t)x_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N_u} c_{i,j}(t)x_j(t) + \lambda_{b,i}, \quad (1)$$

where $\lambda_{b,i} \geq 0$ is the noise due to background radiation. $\Upsilon(\cdot)$ in Fig. 1 denotes the nonlinear transformation operation such that, $y_i(t) = \Upsilon(\mathscr{A}_i), i \in \{1,2,\cdots,N_u\}$ [9]. $\{c_{i,j}(t), t \geq 0\} \in \mathbb{R}_0^+$ is the channel response between the $j$th transmitter and the $i$th receiver. We consider log-normal distributed weak turbulence fading. We assume block-fading channel such that, for any variable $l \in \mathbb{Z}_0^+$, $c_{i,j}(t) = c_{i,j}[l], t \in ((l-1)T_c, lT_c]$, where $T_c$ is the channel coherence time [11].

Let $n_p^{(i)}$ be the number of photons arriving at the $i$th receiver. $n_p^{(i)}$ follows the Poisson distribution with probability given by [12]

$$\begin{aligned} &\Pr\left[y_i(t+\tau) - y_i(t) = n_p^{(i)} \,|x_1(0,T_x), \cdots, x_{N_u}(0,T_x)\right] \\ &= \frac{1}{n_p^{(i)}!}\Theta_i^{n_p^{(i)}} \exp\{-\Theta_i\} \end{aligned}, \quad (2)$$

where $\Theta_i$ is given by

$$\Theta_i = \int_t^{t+\tau} \left( c_{i,i}\left(t'\right) x_i\left(t'\right) + \sum_{\substack{j=1 \\ j \neq i}}^{N_u} c_{i,j}\left(t'\right) x_j\left(t'\right) + \lambda_{b,i} \right) dt' \tag{3}$$

We define the signal-to-noise-plus-interference ratio (SINR) for the $i$th user as a ratio of the squared expected mean of the $i$th user signal component to the total variance of the signal received at the $i$th user [13]. It yields the expression of SINR for the $i$th user, $i \in \{1,2,\cdots,N_u\}$ as shown in (4).

## III. Game Formulation

Considering the randomness of the propagation channel, we formulate the game with incomplete information as follows.

- $\mathscr{N}$ represents a finite set of $N_u$ players, indexed by $i$, such that $i \in \mathscr{N} = \{1,2,\cdots,N_u\}$.
- $\Lambda = \Lambda_1 \times \Lambda_2 \times \cdots \times \Lambda_{N_u}$ denotes a finite set of actions. $\Lambda_i, i \in \{1,2,\cdots,N_u\}$, is the set of actions corresponding to the user $i$ with action profile represented by a vector $\Delta = (\Delta_1, \Delta_2, \cdots, \Delta_{N_u})$, where $\Delta_i \in \Lambda_i$ represents a typical action of the $i$th user.
- $\mathscr{T} = \mathscr{T}_1 \times \mathscr{T}_2 \times \cdots \times \mathscr{T}_{N_u}$ represents the type space of the game with $t_i \in \mathscr{T}_i$ being the typical type space of the $i$th user.
- $\mathscr{U} = (\mathscr{U}_1, \mathscr{U}_2, \cdots, \mathscr{U}_{N_u})$ where $\mathscr{U}_i(\Delta;t), i \in \{1,2,\cdots,N_u\}$ is the payoff for the $i$th user corresponding to the set of action $\Delta$ and type $t$.

We consider three different scenarios for performance analysis.

**Case I**: $c_{i,j} < c_{Th}, \ \ \forall j \in \mathscr{N} \setminus \{i\}$.
It corresponds to a transmission scenario where the interference from the other users are minimum. Case I results in a utility as depicted in (5).

$$\begin{aligned} \mathscr{U}_i &= R_I - F\left(\delta P_{t,i}\right) \\ &= \delta\left(P_{t,i} - \lambda_{b,i}\right) \log\left(1 + \frac{P_{t,i}}{\lambda_{b,i}}\right) - F\left(\delta P_{t,i}\right) - 1 \end{aligned} . \tag{5}$$

where $R_I$ is the throughput that corresponds to Case I. We like to point out that $R_I$ is obtained under the limiting conditions where $\delta P_{t,i} \to 0$ and can readily be derived by utilizing the concept of capacity per unit cost [14]. $F(\cdot) \longmapsto \mathbb{R}$ is the cost function.

The average payoff for Case I can then be obtained as depicted in (6).

TABLE I
SIMULATION PARAMETERS FOR MULTIUSER INDOOR SYSTEM

| Parameter | Value |
|---|---|
| Modulation bandwidth | 20 MHz |
| Active area of the receiver | 1 $\text{cm}^2$ |
| Noise photon count rate $\Theta_{j,b}$ | 14500 $\text{sec}^{-1}$ |
| Peak-to-average power ratio $\delta$ | 0.5 |
| PMT quantum efficiency | 0.30 |

**Remark.** *We like to point out that $s_j(\boldsymbol{c}_j), \forall j \in \mathscr{N}$ represents the strategy of the $j$th user and is a function of the vector $\boldsymbol{c}_j = (c_{j,1}, c_{j,2}, \cdots, c_{j,N_u})$.*

**Case II**: $c_{i,j} \geq c_{Th}, \ \ \forall j \in \mathscr{N} \setminus \{i\}$ and $\chi_i \geq \chi_{Th}$.
It corresponds to the strong interference region. In this case, all the users $j \in \mathscr{N} \setminus \{i\}$ produce interference to the $i$th user. The best strategy would be to transmit at a lower data rate. The corresponding utility is illustrated in (7) [15].

In obtaining $\mathscr{U}_i$, we consider strong interference Poisson channel [15]. As depicted in [15], $p_i$ is the probability of the binary input given by $p_i = \Pr[x_i = 1] = 1 - \Pr[x_i = 0]$, and, for any $j, j \in \mathscr{N} \setminus \{i\}$, the following conditions need to be satisfied for (7) to hold true

$$\frac{c_{i,j}}{c_{j,j}} \geq \frac{\lambda_{b,i}}{\lambda_{b,j}} \geq \frac{c_{i,i}}{c_{j,i}}, \tag{8}$$

and

$$\frac{c_{i,j}}{c_{j,j}} \geq 1 \geq \frac{c_{i,i}}{c_{j,i}}. \tag{9}$$

Utilizing (7), the average payoff for Case II can be expressed as illustrated in (10).

**Case III**: $c_{i,j} \geq c_{Th}, \ \ \forall j \in \mathscr{N} \setminus \{i\}$ and $\chi_i < \chi_{Th}$.

Case III corresponds to a condition where all the interferers have higher channel gain and the SINR of the $i$th user is less than the threshold $\chi_{Th}$. In this scenario, the $i$th user will choose to back off and will not transmit. The utility corresponding to Case III is given by

$$\mathscr{U}_i = -F(\delta P_{t,i}) \tag{11}$$

The average payoff can then be given by the form illustrated in (12).

## IV. Results and Discussions

In this section, we present results to validate the analysis presented in the previous sections.

The adverse impact of increasing the threshold $\chi_{Th}$ on the performance is illustrated in Fig. 2. As depicted in the figure, the transmission probability $\Pr[c > c_{Th}]$, (as defined

$$\chi_i = \frac{\left(\Upsilon\left(E\left[x_i(t)\right]E\left[c_{i,i}(t)\right]\right)\right)^2}{2\left(\Upsilon\left(E\left[x_i(t)\right]E\left[c_{i,i}(t)\right]\right)\right) + 2\left(\Upsilon\left(\sum_{\substack{j=1\\ j\neq i}}^{N_u} E\left[x_j(t)\right]E\left[c_{i,j}(t)\right]\right)\right) + 4\lambda_{b,i}}. \tag{4}$$

$$\begin{aligned} & E\left[\mathscr{U}_i\left(s_i = R_I, s_1\left(c_{i,1}\right), \cdots, s_{i-1}\left(c_{i,i-1}\right), \cdots, s_{i+1}\left(c_{i,i+1}\right), c_{i,i}\right)\right] \\ & = \prod_{\substack{j=1\\ j\neq i}}^{N_u} \Pr\left[c_{j,i} < c_{Th}\right]\left(R_I - F\left(\delta P_{t,i}\right)\right) \end{aligned} . \tag{6}$$

$$\begin{aligned} \mathscr{U}_i = \min\Bigg\{ & E\left[\Omega\left(c_{i,i}P_{t,i}x_i, \sum_{\substack{j=1\\ j\neq i}}^{N_u} c_{i,j}P_{t,j}x_j; c_{i,i}P_{t,i}p_i, \lambda_{b,i}\right)\right], \\ & E\left[\Omega\left(\sum_{\substack{j=1\\ j\neq i}}^{N_u} c_{j,i}P_{t,i}x_i, \sum_{\substack{j=1\\ j\neq i}}^{N_u} c_{j,j}P_{t,j}x_j; \sum_{\substack{j=1\\ j\neq i}}^{N_u} c_{j,i}P_{t,i}p_i, \sum_{\substack{j=1\\ j\neq i}}^{N_u} \lambda_{b,j}\right)\right]\Bigg\} - F\left(\delta P_{t,i}\right) \end{aligned} . \tag{7}$$

in [10]), decreases with the increase in the threshold $\chi_{Th}$. Higher threshold values restrict the frequent transmission of the data.

The achievable capacity (bit/s/Hz) per user relative to the photons arrival rate is illustrated in Fig. 4. These results are presented for the more general case where we impose strict average- and peak-power constraints and assume a positive dark current $\lambda_{b,i}, \forall i$, equal to 14500 $\text{sec}^{-1}$. In obtaining these results, we set $\delta$ equal to 0.5 and number of users to 4. The results show that, with Nash bargaining, self-interested but cooperative users may achieve a superior performance when compared with Nash equilibrium. As illustrated, the poor performance and inefficiency of the Nash equilibrium, when compared with Nash bargaining is due to the fact that there is no cooperation among the different users present. We like to point out that, for low values of $\delta$, the achievable capacity (bits/s/Hz) is linear in both the power and the bandwidth. However, as the $\delta$ value increases, the achievable capacity becomes logarithmic in power and linear in bandwidth.

With $T_x$ set to 1 ms, the impact of the photons arrival due to the background radiation is illustrated in Fig. 3. Corresponding to ultraviolet spectrum, we consider four different values of $\lambda_b \times T_x$, i.e., $0.01, 0.05, 0.1$, and $0.5$ [3].

## V. Conclusions

We have investigated the problem of selecting the optimal transmission strategy in multiuser photon-counting Poisson block fading channel from a game-theoretic perspective. It has been assumed that the users have no information about the channel gain of the interferers and are only aware of the probability distribution of the channel gain. To account for this incomplete information, we have formulated the problem as a Bayesian game. The optimal strategy for each rational user with conflicting objectives has been obtained. We have analyzed three different

$$
\begin{aligned}
&E\left[\mathscr{U}_i\left(s_i=R_{II}, s_1\left(c_{i,1}\right), \cdots, s_{i-1}\left(c_{i,i-1}\right), \cdots, s_{i+1}\left(c_{i,i+1}\right), c_{i,i}\right)\right] \\
&\quad=\prod_{\substack{j=1 \\ j \neq i}}^{N_u}\left\{\Pr\left[c_{j,i} \geq c_{Th}\right] \Pr\left[\chi_i \geq \chi_{Th} \middle| c_{j,i} \geq c_{Th}\right]\right. \\
&\quad\left.\times E\left[R_{II}-F\left(\delta P_{t,i}\right) \middle| c_{Th} \leq c_{j,i}, \chi_i \geq \chi_{Th}\right]\right\}
\end{aligned} \qquad (10)
$$

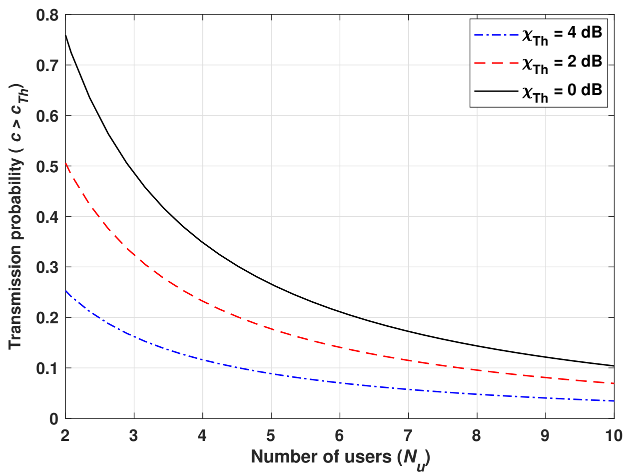


Fig. 2. Transmission probability relative to the number of users for different values of the SINR threshold.

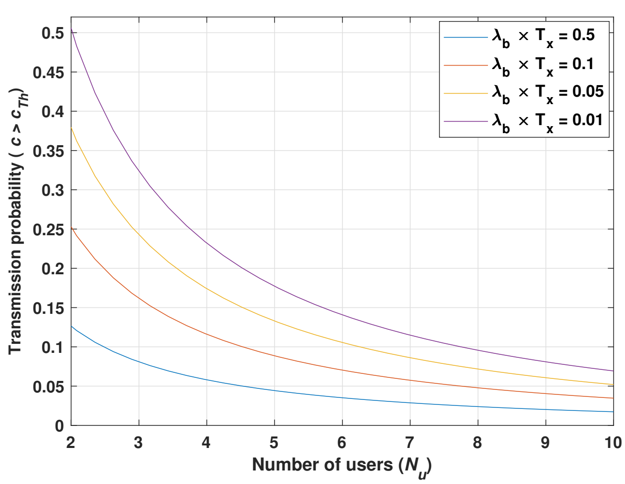


Fig. 3. Transmission probability relative to the number of users for different values of the background radiation.

transmission scenarios. Results are presented to validate the analysis. It can be concluded that with appropriate selection of the transmit strategy, the quality of service can be improved.

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$$
\begin{aligned}
&E\left[\mathscr{U}_i\left(s_i=R_{III}, s_1\left(c_{i,1}\right), \cdots, s_{i-1}\left(c_{i,i-1}\right), s_{i+1}\left(c_{i,i+1}\right), c_{i,i}\right)\right] \\
&= \prod_{\substack{j=1 \\ j\neq i}}^{N_u} \Pr\left[\chi_i < \chi_{Th} \left| c_{j,i} \geq c_{Th}\right.\right]\left(-F\left(\delta P_{t,i}\right)\right) \qquad .
\end{aligned} \tag{12}
$$

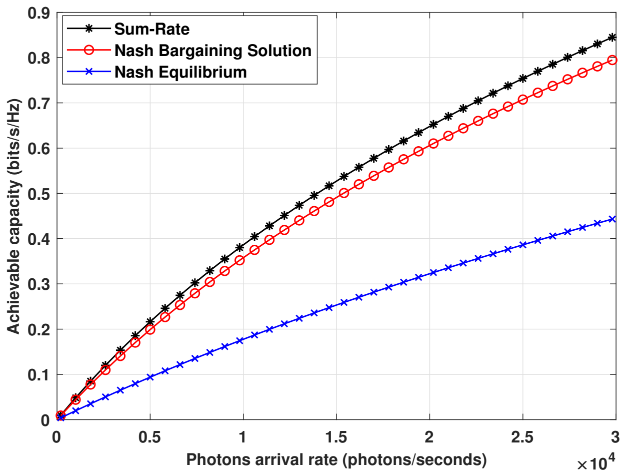


Fig. 4. Achievable capacity relative to the photons arrival rate for different strategies.

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