Joint Impact of Limited Fronthaul and Pilot Length on Payload Data Rate of Cell-Free Massive MIMO

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***Abstract*—This work studies the uplink of a cell-free massive MIMO (mMIMO) system with finite-capacity fronthaul links. In particular, the impacts of fronthaul capacity and pilot length on the channel estimation quality as well as the payload data transmission rates are analyzed. To this end, the powers of the quantization noise signals, the estimated channel coefficients and the estimation errors are calculated first. Then, it is followed by the derivation of the achievable payload data transmission rates under two combining schemes: optimal minimum mean squared error (MMSE) and low-complexity maximum-ratio combining (MRC) schemes. Numerical results show how the average sum-rate of the system is affected by the system parameters such as the fronthaul capacity, uplink signal-to-noise ratio (SNR), and the pilot length.**

***Index Terms*—Cell-free massive MIMO, finite-capacity fronthaul, imperfect CSI.**

I. Introduction

Cell-free massive MIMO system (mMIMO) is envisioned to boost the spectral and energy efficiency of wireless systems by removing cell-edge interference signals [1]–[3]. The advanced interference management of cell-free mMIMO system relies on coherent joint transmission and reception among distributed access points (APs) which are connected to a central processing (CP) unit through fronthaul links. Among essential prerequisites for the joint processing of APs are accurate channel state information (CSI) and reliable fronthaul delivery of baseband signals among APs and CP. A practical channel estimation process and its impact on the overall performance of cell-free mMIMO system were studied in [1]–[3]. Also, the impact of limited fronthaul on cell-free mMIMO system was investigated in [4], [5].

In this work, we study the joint impact of the fronthaul capacity and pilot length on both the channel estimation error and the payload data transmission rates under the uplink of a cell-free mMIMO system. We divide each coherence block, for which the channel fading coefficients remain unchanged, into two phases: channel training and payload data transmission phases. Assuming that user equipments (UEs) transmit orthogonal pilot sequences and each AP adopts uniform scalar quantizer for fronthauling to CP, we calculate the powers of the quantization noise signals, the estimated channel coefficients and the estimation errors. Based on the derived results, we analyze the achievable payload data rates under two combining schemes: optimal minimum mean squared error (MMSE) and low-complexity maximum-ratio combining (MRC) schemes. We investigate the average sum-rate of the two combining schemes for cell-free mMIMO systems with various system parameters via numerical results.

This work was supported in part by the Ministry of Science and ICT under Grant 2021R1C1C1006557; and in part by Basic Science Research Program through the NRF funded by the Ministry of Education under Grant 2019R1A6A1A09031717.

II. System Model

The uplink of a cell-free mMIMO system is considered, in which K single-antenna UEs communicate with a CP through M single-antenna APs. We define $\mathcal{K} = \{1, 2, \ldots, K\}$ and $\mathcal{M} = \{1, 2, \ldots, M\}$ that denote the sets of the indices of UEs and APs, respectively. Each AP is connected to the CP via a digital fronthaul link of capacity C_F bits per symbol.

A. Channel Model and Two-Phase Operation

Assuming flat-fading channel model for the wireless uplink channel from UEs to APs, the received signal y_i of each AP i is modeled as

$$y_i = \sum\nolimits_{k\in\mathcal{K}} h_{i,k} x_k + z_i, \tag{1}$$

where $h_{i,k}$ is the channel coefficient from UE k to AP i, x_k is the transmitted signal by UE k with $\mathbb{E}[|x_k|^2] = P_{\text{tx}}$, and $z_i \sim \mathcal{CN}(0, \sigma_z^2)$ represents the additive noise. The transmit signal-to-noise ratio (SNR) of the uplink channel is defined as $\alpha_{\text{tx}} = P_{\text{tx}}/\sigma_z^2$. We assume that the channel coefficients $h_{i,k}$ are independent across the indices i and k and model each coefficient $h_{i,k}$ as $h_{i,k} \sim \mathcal{CN}(0, \rho_{i,k})$ with the path-loss $\rho_{i,k} = \mathbb{E}[|h_{i,k}|^2]$ given as

$$\rho_{i,k} = \rho_{\text{ref}}(d_{i,k}/d_{\text{ref}})^{-\eta}. \tag{2}$$

Here ρ_{ref} is the path-loss at the reference distance d_{ref}, and η denotes the path-loss exponent.

We consider a block fading model where the channel coefficients $\mathbf{h} = \{h_{i,k}\}_{i\in\mathcal{M},k\in\mathcal{K}}$ remain unchanged for the channel coherence block of during being T symbol periods and change independently from block to block. Each coherence block of T symbols consists of two phases: *i) Channel training phase:* UEs send pilot signals so that the CP estimates the channel coefficients $\mathbf{h}$ based on quantized signals reported from the APs (which will be described in the next subsection); *ii) Payload data phase:* UEs transmit payload data signals to the CP through APs. We assume that for the first phase (i.e., the channel training phase), UEs send orthogonal pilot sequences of length L. Thus, the pilot length L satisfies $K \le L < T$, where $K \le L$ is imposed to guarantee the orthogonality of the pilot sequences, while $L < T$ is required to allow for the payload data transmission.

B. Uniform Scalar Fronthaul Quantizer

Since the fronthaul links are of finite capacity, each AP i sends a quantized version of its received signal y_i to the CP. Assuming a uniform scalar quantizer [6] in which each of in-phase and quadrature (IQ) samples of y_i is quantized with $\lfloor C_F/2 \rfloor$ bits, the quantized version denoted by $\hat{y}_i$ is given as

$$\hat{y}_i = y_i + q_i, \tag{3}$$

where the power of the quantization noise q_i is given as

$$\begin{aligned}\omega_i &= \mathbb{E}\left[|q_i|^2\right] = \frac{3}{2^{C_F}}\mathbb{E}\left[|y_i|^2\right] \\ &= \frac{3}{2^{C_F}}\left(P_{\text{tx}}\sum\nolimits_{k\in\mathcal{K}}\rho_{i,k}+\sigma_z^2\right).\end{aligned} \tag{4}$$

III. Channel Estimation and CSI Error Model

The CP estimates each channel coefficient $h_{i,k}$ based on the quantized signal $\hat{y}_i$ collected from AP i over the channel training phase of L symbol periods. Assuming that the CP performs a linear MMSE (LMMSE) channel estimator, the estimated channel coefficient $\hat{h}_{i,k}$ is related to the actual channel $h_{i,k}$ as

$$h_{i,k} = \hat{h}_{i,k} + e_{i,k}, \tag{5}$$

where $e_{i,k}$ stands for the estimation error uncorrelated to the estimated channel $\hat{h}_{i,k}$ according to the orthogonality principle. The powers of the nominal channel $\hat{h}_{i,k}$ and the estimation error $e_{i,k}$ are given as [2]

$$\mathbb{E}\left[|\hat{h}_{i,k}|^2\right] = \frac{\tilde{\alpha}_{\text{tx},i}L\rho_{i,k}^2}{1+\tilde{\alpha}_{\text{tx},i}L\rho_{i,k}}, \tag{6a}$$

$$\mathbb{E}\left[|e_{i,k}|^2\right] = \frac{\tilde{\rho}_{i,k}}{1+\tilde{\alpha}_{\text{tx},i}L\rho_{i,k}}, \tag{6b}$$

where $\tilde{\alpha}_{\text{tx},i}$ is defined as a modified transmit SNR which accounts for the impact of the quantization noise signal q_i at $\hat{y}_i$, i.e.,

$$\tilde{\alpha}_{\text{tx},i} = \frac{P_{\text{tx}}}{\sigma_z^2+\omega_i}. \tag{7}$$

We can easily check that $\mathbb{E}[|\hat{h}_{i,k}|^2]+\mathbb{E}[|e_{i,k}|^2] = \mathbb{E}[|h_{i,k}|^2]$ as

$$\begin{aligned}&\frac{\tilde{\rho}_{\text{tx},i}L\rho_{i,k}^2}{1+\tilde{\alpha}_{\text{tx},i}L\rho_{i,k}} + \frac{\tilde{\rho}_{i,k}}{1+\tilde{\alpha}_{\text{tx},i}L\rho_{i,k}} \\ =&\frac{\rho_{i,k}}{1+\tilde{\alpha}_{\text{tx},i}L\rho_{i,k}}\left(\tilde{\alpha}_{\text{tx}}L\rho_{i,k}+1\right) = \rho_{i,k} = \mathbb{E}\left[|h_{i,k}|^2\right].\end{aligned} \tag{8}$$

The expressions in (6) show that as the fronthaul capacity has a larger capacity C_F, the power of the estimated coefficient $\hat{h}_{i,k}$ increases, which means that $\hat{h}_{i,k}$ becomes closer to the actual channel $h_{i,k}$. In contrast, the power of the estimation error $e_{i,k}$ decreases with C_F.

IV. Linear Decoding and Achievable Rates

The CP decodes the signals $x_1, x_2, \ldots, x_K$ transmitted by the UEs based on the quantized signals $\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_M$ received on the fronthaul links during the data transmission phase. If we define a vector $\hat{\mathbf{y}} \in \mathbb{C}^{M\times 1}$, which collects all the quantized signals, as $\hat{\mathbf{y}} = [\hat{y}_1\, \hat{y}_2\, \cdots\, \hat{y}_M]^T$, it can be written as

$$\hat{\mathbf{y}} = \sum\nolimits_{k\in\mathcal{K}}\hat{\mathbf{h}}_k x_k + \sum\nolimits_{k\in\mathcal{K}}\mathbf{e}_k x_k + \mathbf{z} + \mathbf{q}, \tag{9}$$

where we have defined the vectors $\hat{\mathbf{h}}_k = [\hat{h}_{1,k}\, \hat{h}_{2,k}\, \cdots\, \hat{h}_{M,k}]^T$, $\mathbf{e}_k = [e_{1,k}\, e_{2,k}\, \cdots\, e_{M,k}]^T$, $\mathbf{z} = [z_1\, z_2\, \cdots\, z_M]^T$, and $\mathbf{q} = [q_1\, q_2\, \cdots\, q_M]^T$. The second term $\sum_{k\in\mathcal{K}}\mathbf{e}_k x_k$ indicates the noise caused by the inaccurate CSI and is regarded as additive noise. Thus, the quantized signal vector $\hat{\mathbf{y}}$ in (9) can be rewritten as

$$\hat{\mathbf{y}} = \sum\nolimits_{k\in\mathcal{K}}\hat{\mathbf{h}}_k x_k + \tilde{\mathbf{z}}, \tag{10}$$

where the effective noise vector $\tilde{\mathbf{z}} = \sum_{k\in\mathcal{K}}\mathbf{e}_k x_k + \mathbf{z} + \mathbf{q}$ is distributed as $\tilde{\mathbf{z}} \sim \mathcal{CN}(\mathbf{0}, \tilde{\mathbf{Z}})$ with

$$\tilde{\mathbf{Z}} = \sum\nolimits_{k\in\mathcal{K}}\text{diag}\left(\left\{\frac{P_{\text{tx}}\tilde{\rho}_{i,k}}{1+\tilde{\alpha}_{\text{tx},i}L\rho_{i,k}}\right\}_{i\in\mathcal{M}}\right) + \sigma_z^2\mathbf{I} + \mathbf{\Omega}, \tag{11}$$

and $\mathbf{\Omega} = \text{diag}(\{\omega_i\}_{i\in\mathcal{M}})$.

To minimize the decoding latency, we assume that the CP performs a parallel linear decoding of the data signals $x_1, x_2, \ldots, x_K$. That is, to decode each data signal x_k, the CP first performs a linear combining as

$$r_k = \mathbf{u}_k^H\hat{\mathbf{y}}, \tag{12}$$

with a combining vector $\mathbf{u}_k \in \mathbb{C}^{M\times 1}$, and then decodes the signal x_k based on the combining output r_k. For given $\mathbf{u}_k$, the achievable data rate R_k of UE k is given as

$$\begin{aligned}R_k &= \left(1-\frac{L}{T}\right)I\left(x_k; r_k\right) \\ &= \left(1-\frac{L}{T}\right)\log_2\left(1+\frac{P_{\text{tx}}|\mathbf{u}_k^H\hat{\mathbf{h}}_k|^2}{\sum_{l\in\mathcal{K}\setminus\{k\}}P_{\text{tx}}|\mathbf{u}_k^H\hat{\mathbf{h}}_l|^2 + \mathbf{u}_k^H\tilde{\mathbf{Z}}\mathbf{u}_k}\right).\end{aligned} \tag{13}$$

Here the factor $(1-L/T)$ comes from the fact that the UEs cannot transmit payload data during the channel training phase whose portion in the time resource is equal to L/T. From the rate expression (13), we can see that the pilot length L needs to be carefully chosen considering its conflicting impacts on the multiplication factor $(1-L/T)$ and the signal-to-interference-plus-noise ratio (SINR) inside the log function.

A. MMSE Combining

As shown in (13), the data rate R_k is affected by the choice of the combining vector $\mathbf{u}_k$. We compare two different choices: the optimal MMSE combiner $\mathbf{u}_k^{\text{MMSE}}$ and a low-complexity maximum-ratio combiner (MRC) $\mathbf{u}_k^{\text{MRC}}$. The optimal MMSE combiner $\mathbf{u}_k^{\text{MMSE}}$ is given such that the SINR at the combiner output, and hence the rate R_k, is maximized. The MMSE combiner $\mathbf{u}_k^{\text{MMSE}}$ is given as

$$\begin{aligned}\mathbf{u}_k^{\text{MMSE}} &= \arg\max_{\mathbf{u}_k\in\mathbb{C}^{M\times 1}}\frac{P_{\text{tx}}|\mathbf{u}_k^H\hat{\mathbf{h}}_k|^2}{\sum_{l\in\mathcal{K}\setminus\{k\}}P_{\text{tx}}|\mathbf{u}_k^H\hat{\mathbf{h}}_l|^2 + \mathbf{u}_k^H\tilde{\mathbf{Z}}\mathbf{u}_k} \\ &= \left(P_{\text{tx}}\sum\nolimits_{l\in\mathcal{K}}\hat{\mathbf{h}}_l\hat{\mathbf{h}}_l^H + \tilde{\mathbf{Z}}\right)^{-1}\hat{\mathbf{h}}_k.\end{aligned} \tag{14}$$

Although the above MMSE combiner provides the best performance, acquiring the MMSE filter $\mathbf{u}_k^{\text{MMSE}}$ is highly complicated particularly for a ultra-dense network with a large number of APs M, since the asymptotic complexity of computing the inverse matrix in (14) is given as $\mathcal{O}(M^3)$ with the standard Gauss-Jordan elimination method.

B. MRC Combining

A simpler well-known choice of $\mathbf{u}_k$ is the MRC combining, which aims at maximizing the ratio of the desired signal power $P_{\text{tx}}|\mathbf{u}_k^H\hat{\mathbf{h}}_k|^2$ to the effective noise power $\mathbf{u}_k^H\tilde{\mathbf{Z}}\mathbf{u}_k$ at the combiner output neglecting the impact of interference signals $\sum_{l\in\mathcal{K}\setminus\{k\}}P_{\text{tx}}|\mathbf{u}_k^H\hat{\mathbf{h}}_l|^2$. The MRC combiner $\mathbf{u}_k^{\text{MRC}}$ is given as

$$\mathbf{u}_k^{\text{MRC}} = \arg\max_{\mathbf{u}_k\in\mathbb{C}^{M\times 1}}\frac{P_{\text{tx}}|\mathbf{u}_k^H\hat{\mathbf{h}}_k|^2}{\mathbf{u}_k^H\tilde{\mathbf{Z}}\mathbf{u}_k} = \tilde{\mathbf{Z}}^{-1/2}\hat{\mathbf{h}}_k. \tag{15}$$

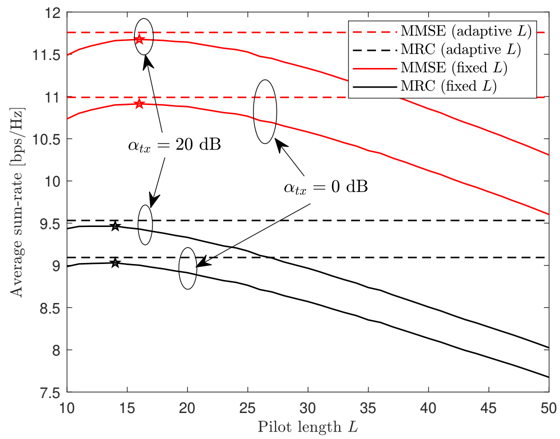

Fig. 1. Average sum-rate versus the pilot length L with varying SNR levels $\alpha_{\text{tx}} \in \{0, 20\}$ dB ($M = 20$ APs, $K = 10$ UEs, and $C_F = 2$ bit/symbol, and $T = 200$ symbols)

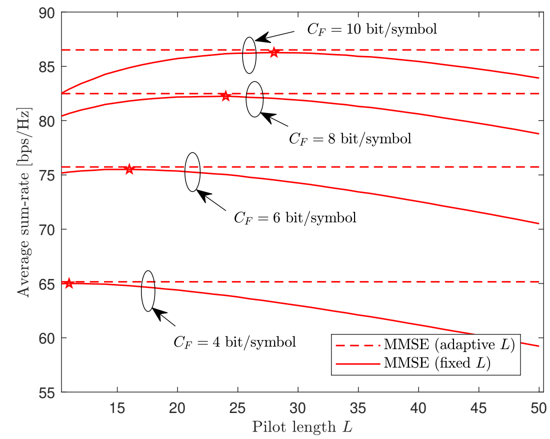

Fig. 2. Average sum-rate versus the pilot length L with varying fronthaul capacity $C_F \in \{4, 6, 8, 10\}$ bit/symbol ($M = 20$ APs, $K = 10$ UEs, $\alpha_{\text{tx}} = 15$ dB, and $T = 300$ symbols)

Here pre-multiplying with $\tilde{\mathbf{Z}}^{-1/2}$ is required to whiten the effective noise signals across the APs. Although the computation of the MRC combiner in (15) also requires an $M \times M$ inverse matrix, the asymptotic complexity is given as $\mathcal{O}(M)$, since $\tilde{\mathbf{Z}}$ is a diagonal matrix as shown in (11).

V. Numerical Results

This section presents numerical results that shows the impacts of pilot length L on the average sum-rate of the uplink of cell-free massive MIMO system with various levels of the transmit SNR α_{tx} and the fronthaul capacity C_F. It is assumed that the UEs and APs are independently sampled from the uniform distribution within a circular region of radius 100 m. For given locations, the path-loss $\rho_{i,k}$ is set according to (2) with $d_{\text{ref}} = 30$ m, $\rho_{\text{ref}} = 10$, and $\eta = 3$.

In Fig. 1, we plot the average sum-rate of both the MMSE and MRC combing schemes while increasing the pilot length L for a cell-free mMIMO system with $M = 20$ APs, $K = 10$ UEs, $C_F = 2$ bit/symbol, transmit SNR levels of $\alpha_{\text{tx}} \in \{0, 20\}$ dB, and $T = 200$ symbols. The solid lines are simulated with fixed pilot lengths $L \in \{10, 11, \ldots, 50\}$ regardless of the path-loss values $\boldsymbol{\rho} = \{\rho_{i,k}\}_{i \in \mathcal{M}, k \in \mathcal{K}}$, while the dashed lines are the average sum-rates achieved when the best pilot length L corresponding to the largest sum-rate is chosen within the range $L \in \{10, 11, \ldots, 50\}$ for each realization of the path-loss $\boldsymbol{\rho}$. The first lesson from the figure is that a dynamic choice of L in adaptation to the path-loss $\boldsymbol{\rho}$ shows a negligible gain compared to the case where L is predetermined to the best value. Comparing the performance of the two combining schemes, the gain of the optimal MMSE combiner is more significant for a high SNR level for which the impact of interference signals is more pronounced. Also, the optimal pilot length is larger for the MMSE combining scheme than for the MRC scheme. This suggests that advanced interference management with the optimal MMSE combining asks for a more accurate CSI at a sacrifice of the pre-factor $(1 - L/T)$.

In Fig. 2, we plot the average sum-rate of the MMSE combining scheme versus the pilot length L for a cell-free mMIMO system with $M = 20$ APs, $K = 10$ UEs, $\alpha_{\text{tx}} = 15$ dB, the fronthaul capacity values of $C_F \in \{4, 6, 8, 10\}$ bit/symbol, and $T = 300$ symbols. As expected, the sum-rate performance is improved as the fronthaul capacity C_F increases, because the fronthaul quantization noise powers $\boldsymbol{\omega} = \{\omega_i\}_{i \in \mathcal{M}}$, which degrade the quality of both the estimated CSI $\hat{\mathbf{h}} = \{\hat{h}_{i,k}\}_{i \in \mathcal{M}, k \in \mathcal{K}}$ and the linear combining output $\mathbf{r} = \{r_k\}_{k \in \mathcal{K}}$, decreases with C_F. Moreover, it is interesting to see that the average sum-rate is maximized for a larger pilot length L as the fronthaul capacity C_F increases. This is because as C_F gets larger, the degradation from the quantization noise becomes less dominated, and it is more important to handle interference with accurate CSI.

VI. Conclusion

We have investigated the joint impact of the fronthaul capacity and pilot length on both the channel estimation error and the payload data transmission rates under the uplink of a cell-free mMIMO system. To this end, we derived the powers of the quantization noise signals, the estimated channel gains, and the estimation errors, and presented relevant numerical results. As future work, we consider an extension to a cell-free mMIMO system with multi-antenna APs.

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