Enhanced Velocity Estimation Based on Joint Doppler Frequency and Range Rate Measurements

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***Abstract*—Modern commercial radars estimate the velocity of target by measuring its Doppler frequency. However, there is a fundamental limit in the detectable Doppler frequency due to Nyquist sampling theorem. The velocity exceeding the maximum detectable velocity is aliased and folded back into the unambiguous detection region, which is known as velocity ambiguity. To address this issue, we propose a novel velocity disambiguation method which combines two velocity estimates with different properties. The first estimate is based on conventional Doppler frequency estimation method which is ambiguous but has high accuracy. The second estimate utilizes the range rate measured for multiple frames which is less accurate but unambiguous. These two estimates are combined to produce a single estimate that is accurate and unambiguous. Simulation results verified that the proposed method can successfully resolve the velocity ambiguity for every velocity values of interest.**

***Index Terms*—Automotive radar, frequency-modulated continuous-wave (FMCW) radar, linear regression, velocity disambiguation.**

I. Introduction

Automotive sensors such as radar, lidar, and camera constitute the advanced driver assistance system (ADAS) and play a major role in autonomous driving [1], [2]. Among the various sensors, the radar sensor is indispensable due to its capability to measure the velocity of targets. The velocity estimation of radar is based on transmitting a radio signal and measuring its Doppler frequency. However, the Nyquist sampling theorem places an upper limit in the maximum detectable velocity and velocities higher than the Nyquist rate are aliased. This phenomenon, which is known as velocity ambiguity, is especially severe in multiple-input multiple-output (MIMO) systems where multiplexing is performed by dividing time slots [3]–[6]. Because velocity estimation is a fundamental function of the radar, the velocity ambiguity needs to be resolved for reliable autonomous driving.

To address this issue, the studies in [7]–[10] have attempted to extend the maximum detectable velocity of MIMO radar systems. However, these methods are limited to restoring the original maximum detectable velocity distorted by MIMO system, and the velocities higher than the Nyquist rate are still aliased. Moreover, these approaches cannot be applied to an arbitrary radar system that does not use MIMO architecture.

Some research have focused on modifying the transmitted signal waveform to resolve velocity ambiguity [11]–[14]. The authors in [11], [12] proposed a velocity disambiguation method by adding a frequency shift between adjacent chirps. In addition, the signals with various repetition intervals were used in [13], [14] and the Chinese remainder theorem was used to extend the maximum detectable velocity. However, the major drawback associated to these methods is that the existing hardware has to be modified.

Another approach utilizes the range estimation results of the radar system [9], [15]. In [9], a rough velocity estimate was obtained using the range variation between adjacent frames and hypothetical test was applied to estimate the unambiguous velocity. Moreover, in [15], a Kalman filter was used to resolve velocity ambiguity by tracking the movement of targets. However, these methods are based on range variation between two adjacent frames, and the performance is deteriorated when the range variation is not large enough to be detected by the radar.

Therefore, in this paper, we propose an enhanced velocity disambiguation method in an frequency-modulated continuous-wave (FMCW) radar system which overcomes the limitations of the aforementioned methods. The proposed method is based on combining the Doppler frequency-based velocity estimate and the range rate-based velocity estimate. The velocity estimate based on the Doppler frequency is ambiguous but highly accurate, whereas the one based on the range rate is less accurate but unambiguous. Therefore, these two velocity estimates are combined to exploit the advantage of each estimate, resulting in an accurate and unambiguous estimate.

The main contributions of the proposed method include: 1) the capability to resolve velocity ambiguity whether the radar system is MIMO or not, 2) the applicability to the existing hardware of the radar system without the need to modify signal waveform, and 3) the use of range information for multiple frames to enhance the range rate measurement capability. To the best of our knowledge, velocity disambiguation utilizing range information for multiple frames has not been sufficiently investigated in the related literature. In addition, we demonstrated that the use of range measurements for five frames is sufficient for velocity disambiguation, which corresponds to accumulating the data for 50 ms. Therefore, the proposed method can be successfully implemented in commercial automotive radar systems and assist ADAS functions such as adaptive cruise control.

The remainder of this paper is organized as follows. In Section II, the principles of FMCW radar system are described and the velocity ambiguity issue is discussed. Next, in Section III, the proposed method of velocity disambiguation is presented. In Section IV, the performance of the proposed method is evaluated using simulation data. Finally, we conclude the paper in Section V.

II. Velocity Estimation in FMCW Radar System

A. Principles of FMCW radar system

The FMCW radar system uses a linear frequency modulation scheme in which the phase of the signal is a quadratic function of time. The transmitted chirp signal can be expressed as

$$s(t) = \cos\left\{2\pi\left(f_c t + \frac{K}{2}t^2\right)\right\} \quad (0 \le t \le T_c), \tag{1}$$

where f_c, K, and T_c denote the carrier frequency, frequency slope, and duration of the transmitted chirp signal, respectively. As shown in Fig. 1, the transmission is repeated for L consecutive chirps. A set of L chirps transmitted in one cycle is denoted as a frame, and each frame is used to estimate the range and velocity of the target.

The transmitted signal is reflected by scatterers in the detection range of the radar, and the delayed echo signals are received at the receive antenna. Then, the received signal is mixed with the transmitted signal and low-pass filtering is applied to the mixed signal. Using quadrature demodulation, the received signal after sampling can be expressed in exponential form as [16]

$$\begin{aligned} x[m,l] \simeq \exp\{j2\pi(f_b m T_s + f_d l T_l + \phi)\} \\ (m = 0, 1, \cdots, M-1) \\ (l = 0, 1, \cdots, L-1), \end{aligned} \tag{2}$$

which is commonly denoted as the beat signal. Here, T_s and T_l are the duration between adjacent samples and chirps, ϕ is the residual phase term, and M is the number of samples. In addition, f_b and f_d are the frequencies along the m-axis and l-axis, respectively, which are denoted as the beat frequency and Doppler frequency.

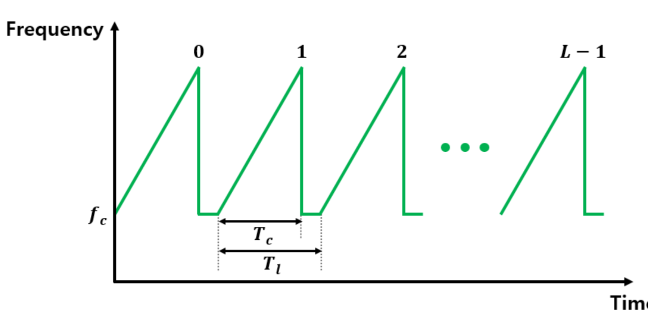


Fig. 1: Transmitted chirp signal of FMCW radar system

The beat frequency and Doppler frequency can be expressed as a function of range and velocity as follows:

$$\begin{aligned} f_b &= \frac{2KR}{c}, \\ f_d &= \frac{2v}{\lambda}, \end{aligned} \tag{3}$$

where R and v are the range and velocity of the target, and λ is the wavelength of the signal. These two frequencies can be estimated by applying two-dimensional (2D) fast Fourier transform (FFT) to the beat signal in (2). Therefore, the range and velocity of the target can be estimated as $\hat{R} = \frac{c}{2K}\hat{f}_b$ and $\hat{v} = \frac{\lambda}{2}\hat{f}_d$, respectively.

B. Velocity ambiguity of FMCW radar system

The estimation of velocity is based on the Doppler frequency of the target, as explained in II-A. However, according to the Nyquist sampling theorem, the sampling frequency places an upper limit on the maximum detectable frequency. Because the sampling frequency along the l-axis is equal to $\frac{1}{T_l}$, the detectable Doppler frequency region is $|f_d| \le \frac{1}{2T_l}$. This translates into the detectable velocity region of $|v| \le \frac{\lambda}{4T_l}$. By defining v_{max} as $\frac{\lambda}{4T_l}$, the unambiguous detection region of velocity is $[-v_{max}, v_{max}]$ [17].

Due to the 2π-periodicity of phase, the velocity outside of unambiguous detection region is folded back into the unambiguous detection region. Fig. 2 shows an example of velocities outside of unambiguous detection region that are falsely estimated due to aliasing. Because the estimated velocity always lies in $[-v_{max}, v_{max}]$, the estimated velocity can be expressed as

$$\begin{aligned} \hat{v} &= v_{true} - \left\lfloor \frac{v_{true} + v_{max}}{2v_{max}} \right\rfloor \times 2v_{max} \\ &= v_{true} - \mathcal{N} \times 2v_{max}, \end{aligned} \tag{4}$$

where v_{true} is the true velocity, $\lfloor \cdot \rfloor$ is the floor function, and $\mathcal{N}$ is an integer value that determines the region depicted in Fig. 2. The integer value $\mathcal{N}$ plays a significant role in velocity disambiguation and will be denoted as the "ambiguity parameter."

The true velocity can be determined from the ambiguously estimated velocity $\hat{v}$ by rearranging the terms in (4),

$$v_{true} = \hat{v} + \mathcal{N} \times 2v_{max}. \tag{5}$$

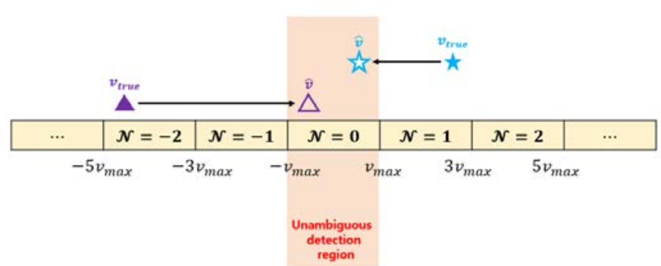


Fig. 2: Velocity ambiguity and unambiguous detection region

Therefore, the velocity ambiguity can be resolved if the ambiguity parameter $\mathcal{N}$ is determined. The method of estimating the ambiguity parameter by combining two velocity estimates will be discussed in Section III.

The velocity ambiguity issue becomes particularly significant when multiple transmit antennas are used for transmission. For example, when the time division multiplexing method is used for MIMO transmission, the transmission is performed alternately by dividing time slots. The duration between homogeneous chirps is increased by the number of transmit antennas, and the unambiguous detection region decreases accordingly. Because the MIMO architecture is being widely used in the radar industry, it is necessary to resolve velocity ambiguity and increase the maximum detectable velocity of the radar.

III. Proposed method of velocity disambiguation

In this section, a method of resolving velocity ambiguity in an FMCW radar system is presented. The overall block diagram of velocity disambiguation is shown in Fig. 3. In the proposed framework, two velocity estimates are obtained from the beat signal in (2). One estimate is based on the Doppler frequency which is ambiguous but highly accurate. The other estimate is based on the range rate which is less accurate but unambiguous. By combining these two estimates, the unambiguous velocity of the target can be estimated with high accuracy.

The proposed method of velocity disambiguation consists of three stages. In the first stage, an ambiguous velocity $\hat{v}$ is estimated by applying FFT along the l-axis in (2) and extracting the Doppler frequency. The ambiguously estimated velocity always lies in the interval $[-v_{max}, v_{max}]$, as explained in Section II-B. Fig. 4 shows an example of an ambiguously estimated velocity and several candidate velocities. It can be observed from the figure that ambiguity parameter $\mathcal{N}$ is needed to uniquely determine the target's velocity among the candidate velocities.

In the next stage, an unambiguous velocity $\dot{v}$ is estimated by using the range rate measured for multiple frames. By applying FFT along the m-axis in (2) and extracting the beat frequency, the range of the target can be estimated. This process is repeated for N_f consecutive frames. Then, the estimated range at k_{th} frame can be expressed as

$$\hat{R}_k = T_f \dot{v} k + C \quad (k = 0, 1, \cdots, N_f - 1), \tag{6}$$

where T_f is the duration of a frame and C is the initial range value. Therefore, a linear relation holds and the range rate $\dot{v}$ can be estimated by applying linear regression algorithms. Among the linear regression algorithms, we employed the LS method which minimizes the sum of squared errors [18]. The N_f equations in (6) can be arranged in matrix form as

$$\underbrace{\begin{bmatrix} 0 & 1 \\ T_f & 1 \\ \vdots & \vdots \\ (N_f-1)T_f & 1 \end{bmatrix}}_{\mathcal{A}} \underbrace{\begin{bmatrix} \dot{v} \\ C \end{bmatrix}}_{\mathcal{X}} = \underbrace{\begin{bmatrix} \hat{R}_0 \\ \hat{R}_1 \\ \vdots \\ \hat{R}_{N_f-1} \end{bmatrix}}_{\mathcal{B}}. \tag{7}$$

The LS solution of the linear equation $\mathcal{AX} = \mathcal{B}$ is given as $\mathcal{X} = A^{\dagger}\mathcal{B} = (\mathcal{A}^T\mathcal{A})^{-1}\mathcal{A}^T\mathcal{B}$, where $(\cdot)^{\dagger}$ is the Moore-Penrose inverse operator. Therefore, $\dot{v}$ can be estimated by calculating $\mathcal{X} = A^{\dagger}\mathcal{B}$ and extracting its first element.

Lastly, two velocity estimates $\hat{v}$ and $\dot{v}$ are combined to determine the ambiguity parameter $\mathcal{N}$ and resolve velocity ambiguity. The ambiguity parameter $\mathcal{N}$ can be determined by finding the candidate velocity that is closest to $\dot{v}$. In other words, $\mathcal{N}$ is determined by solving the following optimization problem,

$$\begin{aligned} \hat{\mathcal{N}} &= \arg\min_{\mathcal{N}} |v_{candidate} - \dot{v}| \\ &= \arg\min_{\mathcal{N}} |\hat{v} + \mathcal{N} \times 2v_{max} - \dot{v}|, \end{aligned} \tag{8}$$

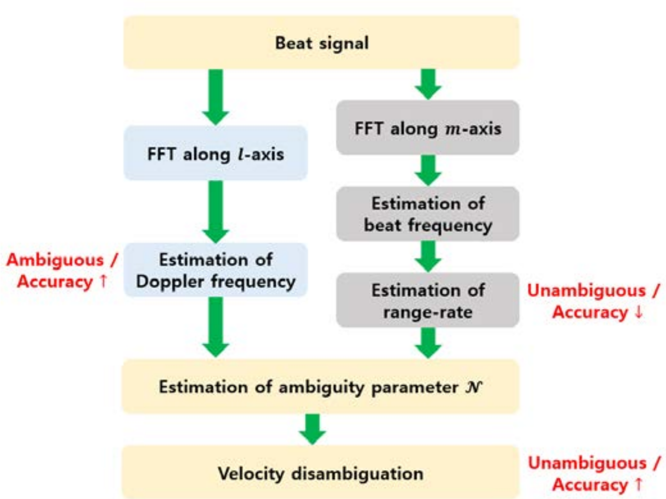


Fig. 3: Block diagram of velocity disambiguation

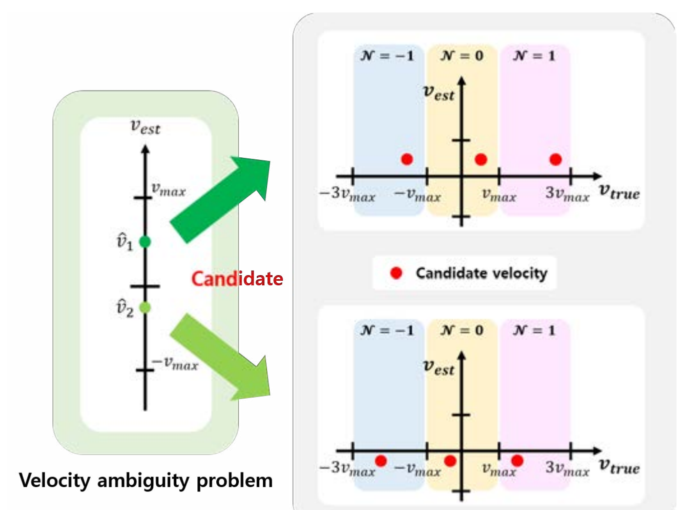


Fig. 4: Candidate velocities according to the estimated velocity

subject to the constraint that $\mathcal{N}$ is an integer. The solution to the above optimization problem can be expressed in closed form as

$$\hat{\mathcal{N}} = \left\lfloor \frac{\dot{v} - \hat{v}}{2v_{max}} + 0.5 \right\rceil. \tag{9}$$

After the ambiguity parameter $\hat{\mathcal{N}}$ is determined, the unambiguous velocity of the target can be calculated using (5).

IV. Performance evaluation

In this section, the performance of the proposed velocity disambiguation method is evaluated using simulated data. The parameters of the FMCW radar system used in simulations are listed in Table I. We assumed a MIMO radar system with two transmit antennas, and the maximum unambiguous velocity was 9.73 m/s. In addition, we assumed that there are three targets in the detection range of the radar, and the range and velocity of targets were set as Table II. Among the three targets, the velocity of target B lies inside the unambiguous detection region, as can be seen from $\mathcal{N} = 0$. On the other hand, the rest of the two targets have velocities outside of the unambiguous detection region, with values of $\mathcal{N} = 1$ and -2, respectively. Therefore, it is expected that the velocities of targets A and C will be folded back into the unambiguous detection region, leading to false estimates.

First, a 2D FFT was applied to the beat signal to simultaneously estimate the range and velocity of targets. The resulting 2D spectrum is shown in Fig. 5. The velocity of target B was correctly estimated as 5.02 m/s because the velocity lies inside the unambiguous detection region. However, the velocities of targets A and C were falsely estimated as -4.41 m/s and -2.59 m/s, respectively. Because of velocity ambiguity, the conventional Doppler frequency estimation method resulted in large error.

Next, the range rate of the target was estimated by accumulating the range estimation results for multiple frames. Fig. 6 shows the range variation of three targets as the frame advances. In the figure, the number of frames was set as 30 to clearly show the range variation according to frame index. However, the number of frames cannot be set to a large value when an immediate response is needed in a dynamic environment. By accumulating more frames for range rate calculation, the estimated velocity will be more accurate, but the computation time required for velocity estimation will also increase. On the other hand, by using fewer frames for range rate calculation, the velocity estimation can be performed quickly but the estimated velocity can be unreliable. The number of frames for calculating range rate is a design parameter that should be set appropriately depending on desired applications. In our work, the number of frames was set as five by considering the tradeoff between performance and complexity, which corresponds to accumulating the data for 50 ms. Because only an approximate velocity estimate is needed when determining the ambiguity parameter $\mathcal{N}$, using five range information was sufficient and resulted in range rate estimates of 20.49 m/s, 0 m/s, and -23.42 m/s, respectively. The values of $\mathcal{N}$ were correctly estimated for all three targets as 1, 0, and -2, respectively, and the unambiguous velocities were reconstructed as 15.06 m/s, 5.02 m/s, and -22.05 m/s.

Furthermore, we compared the performance of the proposed

TABLE I: Parameters of the FMCW radar system

Radar parameter	Value
Carrier frequency, f_c	77 GHz
Frequency slope, K	10 MHz/μs
Sampling period, T_s	0.1 μs
Chirp interval, T_l	50 μs
Number of samples per chirp, M	256
Number of chirps per frame, L	128
Frame duration, T_f	10 ms

TABLE II: Range and velocity of targets

Index	Range	Velocity	$\mathcal{N}$
Target A	30 m	15 m/s	1
Target B	50 m	5 m/s	0
Target C	20 m	-22 m/s	-2

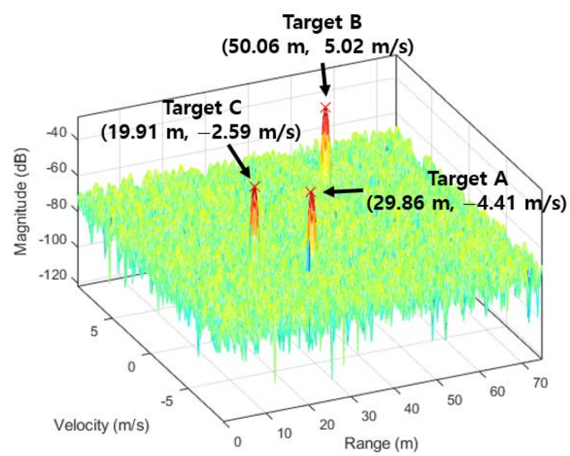


Fig. 5: Target detection result with velocity ambiguity

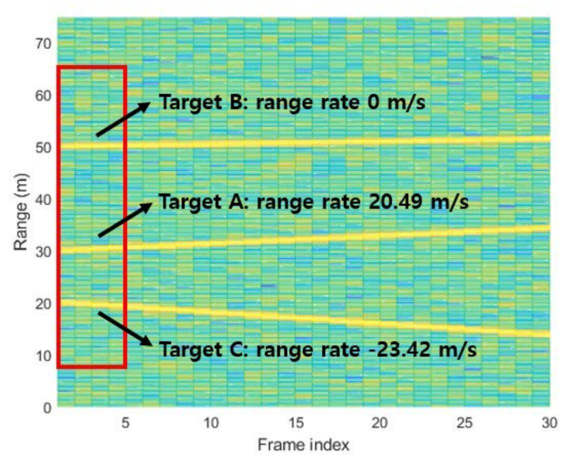
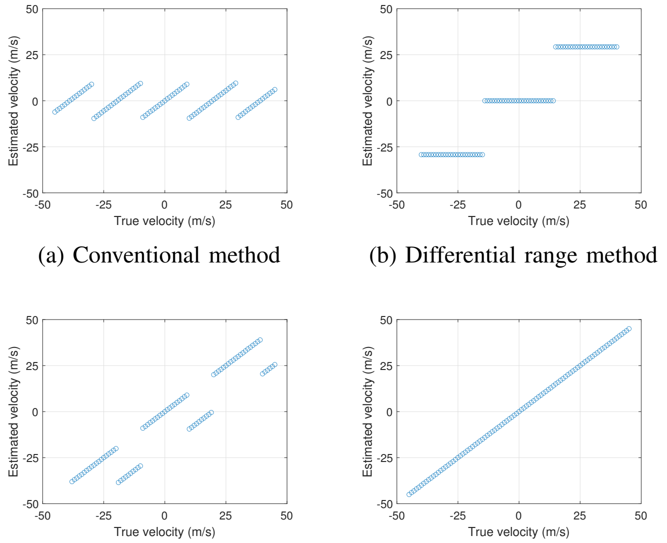


Fig. 6: Range variation of target as frame advances



(a) Conventional method (b) Differential range method

(c) Hypothetical velocity method (d) Proposed method (Frame: 5)

Fig. 7: Comparison of various velocity estimation methods

method with the conventional Doppler frequency estimation method, differential range method, and hypothetical velocity method used in [9]. The velocity of the target was set as $[-5v_{max}, 5v_{max}]$, and the range was set as a random value between 20 m and 80 m. As shown in Fig. 7, the conventional Doppler frequency estimation method resulted in velocity ambiguity and failed to estimate the velocities outside of $[-v_{max}, v_{max}]$. Also, the differential range method which calculates the range variation between adjacent frames resulted in an unreliable estimate. The hypothetical velocity method in [9] which combines the results of differential range method with several hypothetical velocities showed much more stable performance. However, there was an error in some of the velocity values, as can be seen from the discontinuity. This is because the method in [9] only uses the range information of two frames and occasionally selects the false hypothesis when the range rate estimate is inaccurate. In contrast, the proposed method was able to correctly estimate the target's velocity for all values of interest and velocity ambiguity was resolved. Therefore, we believe that the proposed method of velocity disambiguation can help to improve the detection performance of radar by overcoming its hardware limitations.

V. Conclusion

In this paper, a method of resolving velocity ambiguity in an FMCW radar system was proposed. The proposed method was based on combining two velocity estimates, the Doppler-frequency based estimate and the range-rate based estimate. By combining two velocity estimates, the ambiguity parameter was determined and the target's true velocity was correctly estimated. The simulation results verified that the proposed method can successfully resolve velocity ambiguity for all velocity values of interest. Also, we revealed that the use of five range measurements is sufficient for range rate calculation, which demonstrates that the proposed method can be successfully implemented in commercial automotive radar systems.

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