A Low-Complexity Multiuser Adaptive Modulation Scheme for MRC Receivers in Massive MIMO Systems

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***Abstract*— This paper presents a low-complexity adaptive scheme for an uplink multiuser transmission in a massive multiple-input and multiple-output (MIMO) system using maximum-ratio combining (MRC) at the receiver. Instead of using small-scale fast-varying fading, slow-varying large-scale shadowing information is exploited to adapt the desired mode among a discrete set of modulations available at the transmitter side. Since users experience different large-scale fading parameters, the adaptation procedure has to be calculated separately for each user. Therefore, this scheme is called multiuser adaptive modulation (MAM) and is compared to the traditional approach called fast adaptive modulation (FAM), which accounts for small-scale fast-varying fading for adaptation. Furthermore, the closed-form expressions for the average spectral efficiency and bit error outage are derived. In comparison with the FAM scheme and the previous research results, the outcome of this paper shows that at low transmit power, the aforementioned adaptive scheme by the MRC receiver gives a similar performance.**

Keywords*— *Adaptive modulation, massive MIMO, power control, maximum-ratio combining, large-scale fading.

I. Introduction

Two effective methods can improve spectral efficiency, adaptive modulation and power control, regarding the inherent channel fading in wireless communication systems. With a certain quality of service (QoS) requirement, the spectral efficiency can be increased by setting a proper level of modulation or transmit power in the transmitter [1], [2]. Recently, different kinds of adaptive modulation and coding (ACM) schemes have been proposed to achieve higher spectral efficiency as well as improve network reliability for optical networks [3], [4]. In addition, adaptive schemes that exploit various machine learning algorithms have been of interest, aiming to automatically compute decision-making tasks [5]-[7]. According to recently cited studies, the transmitter should access the instantaneous channel state information (CSI). However, according to the variation of wireless channels, pursuing the instantaneous CSI is costly. Therefore, statistical CSI-based design with low complexity has been under attention to evaluate ergodic spectral efficiency in the literature [8], [9]. Hence, an adaptive modulation scheme is desirable for slowly changing large-scale fading [10]-[12].

A low-complexity multiuser adaptive modulation scheme has been investigated in [12], where zero-forcing (ZF) detection is employed at the receiver. Their results suggest multiuser adaptive modulation (MAM) scheme that can achieve similar average spectral efficiency performance as the Fast Adaptive Modulation (FAM) scheme. Moreover, among simple linear detectors, the Minimum Mean Square Error (MMSE) is known to be the best detector that maximizes the received signal-to-interference-plus-noise ratio (SINR). This is because ZF and MMSE have almost identical performances for increased transmission power. At low transmit power, maximum ratio combining (MRC) performs the same as MMSE because any inter-user interference portion can be degraded. While ZF receiver mitigates multiuser interferences by pseudo-inverse channel matrix, it neglects noise effects. Therefore, the noise is significantly amplified by the pseudo-inverse channel matrix when the channel is not well conditioned, leading to performance degradation of ZF receivers [13], [14]. MRC receivers, on the other hand, use a pretty low-complexity algorithm because they do not suffer the implementation complexity since there is no need to inverse the channel matrix. This complexity is so challenging that many researchers have proposed different algorithms to reduce it, especially in massive MIMO systems with large sizeable dimensional matrix inversion [15]-[17].

Table.1 shows the time complexity for detection of a transmitted symbol vector relative to the number of users (n) [16] and [22]. It can be readily seen that the complexity could be alleviated using MRC. In [18], the authors only assumed the transmission under Rayleigh fast-varying flat-fading channel. They have investigated the probability density function (PDF) of SINR

TABLE I. Computational Complexity Comparison

Algorithm	*Time Complexity*
QR decomposition with ZF [16]	$O(\mathbf{n}^{2.529})$
Coppersmith –Winograd with MMSE only [22]	$O(\mathbf{n}^{2.376})$
MRC	$O(\mathbf{n})$

for the first time considering MRC at the receiver. However, large-scale slow-varying fading has not been considered in this study. Furthermore, no network metrics have been derived and investigated.

Motivated by the discussed observations, in this paper, we evaluate a low-complexity multiuser adaptive modulation scheme when MRC is employed at the receiver. To this aim, we derive an analytical expression for the exact distribution of SINRMRC under Rayleigh fast-varying flat-fading along with slow-varying large-scale shadowing conditions. Then, we use exact analytical expressions to calculate the average spectral efficiency (ASE) and bit error outage (BEO). Furthermore, in a low transmit power regime, which is the appropriate case for MRC, the derived expressions are approximated, and the system performance is evaluated. Finally, in the MRC structure, we investigate the effect of users' distances (arrangement) from the base station (BS) on the maximum achievable ASE. The results show that the proposed low-complexity MAM scheme can achieve similar ASE performance as the FAM scheme with even less complexity than [12]. Furthermore, the ASE can be substantially improved by exploiting joint power control.

The rest of this paper is organized as follows. Section II presents the system model used in this paper. Section III describes the MAM scheme. In Section IV, the approximation in low transmit power regime is obtained following the numerical and simulation results presented in Section V. Finally, concluding remarks are depicted in section VI.

II. System Model

A communication system in uplink transmission with a massive MIMO structure containing one BS with M antennas and K single-antenna users in a single cell is considered. The $M \times 1$ received signal at the BS can be expressed as:

$$\boldsymbol{y} = \boldsymbol{G}\boldsymbol{P}^{1/2}\boldsymbol{x} + \boldsymbol{n} \tag{1}$$

where $\mathbf{G} \in C^{M \times K}$ is the channel matrix between the BS antennas and the K users, $\mathbf{P} \triangleq \mathrm{diag}\{p_1, \dots, p_K\}$ where p_k is the average transmit power of user k, $\mathbf{x}$ is the $K \times 1$ transmit signal vector with element x_k satisfying $E\{{x_k}^2\} = 1$ and $\boldsymbol{n}$ is the additive zero-mean white Gaussian noise with a unity variance.

The channel matrix is represented by both large-scale and small-scale fading; hence the channel matrix $\mathbf{G}$ can be expressed as [12]:

$$\boldsymbol{G} = \boldsymbol{H}\boldsymbol{D}^{1/2} \tag{2}$$

where $\mathbf{H} \in \mathrm{C}^{\mathrm{M} \times \mathrm{K}}$ is the fast fading channel matrix consisting of independent and identically distributed (i.i.d) circular complex zero-mean Gaussian variable with unity variance.

$\mathbf{D} = \mathrm{diag}\{\beta_1, \dots, \beta_K\}$, where $\beta_k = \frac{z_k}{\left({r_k}/{r_h}\right)^v}$ is the large-scale fading coefficient that accounts for both path loss and shadowing, $\mathrm{r_k}$ is the distance between the user k and the BS, r_h is the reference distance to the BS, and v is the path loss exponent [12].

It has to be noted that the gamma distribution has been verified as an appropriate choice to model the shadow fading z_k according to [12] and [19].

$$f_{z_k}(x) = \frac{x^{\alpha-1}}{\Gamma(\alpha)b^{\alpha}}\exp\left(-\frac{x}{b}\right) \tag{3}$$

where α, b are shape and scale parameters, respectively. By using linear receivers (MRC) at the BS, r_k as the kth user received signal can be written as:

$$r_k = \sqrt{p}\beta_k\|\boldsymbol{h}_k\|^2 x_k + \sum_{\substack{i=1 \\ i \neq k}}^{K} \sqrt{p\beta_k\beta_i}\boldsymbol{h}_k{}^H\boldsymbol{h}_i x_i + \sqrt{\beta_k}\boldsymbol{h}_k{}^H\boldsymbol{n} \tag{4}$$

For simplicity, equal average transmit powers, p, by users are assumed. Moreover, the term $\sqrt{p}\beta_k\|\boldsymbol{h}_k\|^2 x_k$ represents the desired signal, $\sum_{\substack{i=1 \\ i \neq k}}^{K} \sqrt{p\beta_k\beta_i}\boldsymbol{h}_k{}^H\boldsymbol{h}_i x_i$ represents the multiuser interference (MUI) and $\sqrt{\beta_k}\,\boldsymbol{h}_k{}^H\boldsymbol{n}$ denotes the additive noise. Finally, the SINR for the kth user is given by:

$$\chi_k = \frac{p\,\beta_k\,\|\boldsymbol{h}_k\|^2}{p\sum_{\substack{i=1 \\ i \neq k}}^{K} \beta_i \frac{\left|\boldsymbol{h}_k{}^H\boldsymbol{h}_i\right|^2}{\|\boldsymbol{h}_k\|^2} + 1} \tag{5}$$

To evaluate the MAM scheme, the averaged fast fading SINR ($\bar{\chi}$), with the help of channel hardening property is introduced in the next section. To this aim, the conditional PDF of instantaneous SINR on $\bar{\chi}$ must be described in the next section.

III. Multiuser Adaptive Modulation Scheme (MAM)

In this section, we first investigate a low-complexity MAM scheme proposed by [12] for massive MIMO systems using MRC detection at the BS, then present closed-form and exact expressions for the ASE and BEO.

A. Multiuser Adaptive Modulation Scheme

By recalling the massive MIMO regime, the column vectors of the channel matrix are asymptotically orthogonal, i.e., $\left[\frac{\mathbf{H}^{\mathbf{H}}\mathbf{H}}{M}\right] \xrightarrow{a.s.} \mathbf{I}$, where $\xrightarrow{a.s.}$ denotes almost sure convergence as $M \gg K$. Therefore, the approximated SINR of user k can be evaluated by:

$$\bar{\chi}_k \approx Mp\beta_k \tag{6}$$

We refer to all the above discussions about the properties of approximated SINR variables and selecting different modulation types to the work of [12].

B. SINR Thresholds

Before computing the SINR thresholds, we need to find a conditional PDF on $\bar{\chi}_k$. However, the conditional PDF of the instantaneous SINR can be expressed as follows.

Theorem: *Approximated conditional PDF for the channel state in the case all users experience different shadowing and path loss:*

$$f_{X_k|\bar{X}_k}(\chi|\bar{\chi}) \approx \frac{\left(\frac{M}{\bar{\chi}}\right)^{M} \chi^{M-1} \exp\left(\frac{\bar{\chi}}{2Mp\Omega_k\chi} - \frac{M}{\bar{\chi}}\chi\right)}{(p\Omega_k)^{\frac{\theta_k}{2}}\Gamma(M)\Gamma(\theta_k)} \times \left(\sum_{i=0}^{M}\binom{M}{i}\Gamma(\theta_k+i)\,\Gamma(i+1)\times \dagger_{i,k}(\chi,\bar{\chi})\right) \quad (7)$$

where:

$$\dagger_{i,k}(\chi,\bar{\chi}) = \left(\frac{M\chi}{\bar{\chi}}\right)^{-\frac{1}{2}(\theta_k+2i)} W_{-\frac{1}{2}(\theta_k+2i),\ \frac{1}{2}(\theta_k-1)}\left(\frac{\bar{\chi}}{Mp\Omega_k\chi}\right) \quad (8)$$

$$\theta_k = a\frac{\left(\sum_{i=k,i\neq k}^{K}\lambda_i\right)^2}{\sum_{i=k,i\neq k}^{K}\lambda_i^{\,2}}, \qquad \Omega_k = a\frac{\sum_{i=k,i\neq k}^{K}\lambda_i}{\theta_k} \quad (9)$$

And:

$$\lambda_i = \frac{b}{d_i^{\,v}},\ d_i^{\,v} = \left({}^{r_i}/_{r_h}\right)^{v} \quad (10)$$

Proof: See Appendix.

■

Then the approximated BER as a function of average SINR $\bar{\chi}_k$ is obtained by averaging BER of the instantaneous SINR, $P_b(\chi)$ over the fast fading effect as follows:

$$P_b(\bar{\chi}_k) = \int_0^\infty P_b(\chi) f_{X_k|\bar{X}_k}(\chi|\bar{\chi})\,d\chi \quad (11)$$

We shall consider exact formulas for BER, $P_b(\chi)$ in (11). We can follow integral calculations like [12] to reach a closed-form approximation for BER. Hence, in all cases, the thresholds $\{\bar{x}_{th}^{n}\}_{n=1}^{N+1}$ for MAM with different modulation schemes are obtained by solving $P_b(\bar{x}_{th}^{n}) = P_b^{*}$. Calculations reveal that for a maximum level of modulation, 64-QAM and $K = 5$, by $M \geq 1024$, $BER_{design} = 10^{-3}$, for each users' arrangement, we have a table of SINR thresholds for each user and power value. In other words, in a fixed value of power and a specific users' arrangement or distance vector (DV), a list of SINR thresholds for each user is generated, shown in Fig.1. On the other hand, In the FAM scheme, thresholds could be calculated for the different levels of

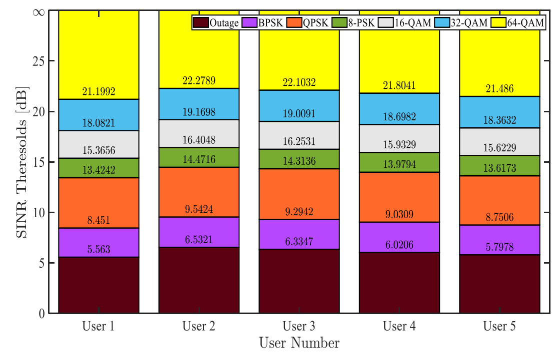

Fig. 1: SINR thresholds at power 0 [dB] for a fixed users' arrangement; DV = [400 450 500 550 600]; M = 2048, K = 5.

modulations by typical bit error rate in an AWGN channel as $P_b(x_{th}^{n}) = P_b^{*}$, [20].

C. Average Spectral Efficiency

Now we compute the ASE of the proposed adaptive modulation scheme. Notably, the spectral efficiency of user k is n if $\bar{\chi}_k$ is in the range of $[\bar{x}_{th}^{n}(p_k,k), \bar{x}_{th}^{n+1}(p_k,k)]$, that $\bar{x}_{th}^{n}(p_k,k)$ show SINR thresholds that depend on power and are different for each user. It can be shown that the ASE of user k can be expressed as:

$$\overline{SE}^{MAM} = \sum_{k=1}^{K}\sum_{n=1}^{N}\frac{n}{\Gamma(a)}\left[\Upsilon\left(a,\frac{\bar{x}_{th}^{n+1}(p_k,k)}{Mp_k\lambda_k}\right) - \Upsilon\left(a,\frac{\bar{x}_{th}^{n}(p_k,k)}{Mp_k\lambda_k}\right)\right] \quad (12)$$

where $\Upsilon(.,.)$ is the incomplete Gamma function [21, eq. (8.350.1)].

D. Bit Error Outage

The Bit Error Outage (BEO) is the probability that the BEP exceeds the target BEP value [11], [12]. In the following, we compute the BEO for both FAM and MAM schemes. For the FAM scheme, the BEO is the probability that the instantaneous value of the SINR falls under a given threshold, while in the MAM scheme, approximated SINR have to be considered:

$$BEO^{FAM} = P(\chi < \bar{x}_{th}^{1}) \quad (13)$$

There is no closed-form solution for the FAM scheme, so it must be calculated numerically.

As mentioned before about the MAM scheme, the transmission of user k will be in an outage if the SINR $\overline{\chi_k}$ falls under the corresponding threshold $\tilde{x}_{th}^{1}(p,k)$. Hence, according to [12], the BEO of user k in all cases is:

$$BEO_k^{MAM} = P(\overline{\chi_k} < \bar{x}_{th}^{1}(p,k)) = \frac{1}{\Gamma(a)}\Upsilon\left(a,\frac{\bar{x}_{th}^{1}(p,k)\,d_k^{v}}{Mpb}\right) \quad (14)$$

It is worth adding that adaptation with joint power control subject to an average transmit power, BER constraints, and the corresponding optimization problem are the same expressions presented by [12].

IV. Approximation under Low Power Levels Conditions

As we can see, the previous expressions for PDFs are so complicated to use for performance evaluation. On the other hand, in massive MIMO systems, MRC receivers are mainly used in lower SNRs [18]. This motivates us to approximate PDFs derived earlier in low power conditions. Finally, the corresponding PDF is approximated by:

$$f_{X_k|\bar{X}_k}(\chi|\bar{\chi}) \approx \frac{\chi^{M-1}\, exp\left(-\frac{M}{\bar{\chi}}\chi\right)}{\Gamma(M)\left(\frac{\bar{\chi}}{M}\right)^{M}} \tag{15}$$

A. SINR Thresholds

It can be seen from BER equations, the SINR threshold values do not depend on power, and they have the same results for all users in low power transmission.

B. Average Spectral Efficiency

The ASE relation in the MAM scheme is identical. Because It depends on the pdf of approximated SINR, $\bar{\chi}$. The overall systems ASE, $\overline{SE}^{FAM}$, regarding the PDF of the instantaneous SINR, can be expressed as:

$$\overline{SE}^{FAM} = \sum_{k=1}^{K}\sum_{n=1}^{N} n\left[\left[\sum_{m=0}^{M-1}\frac{2(x_{th}^{n}d_k^{v})^m}{m!\,\Gamma(a)b^a p^m}\times\left(\frac{x_{th}^{n}d_k^{v}\,b}{p}\right)^{\frac{a-m}{2}} K_{a-m}\left(2\sqrt{\frac{x_{th}^{n}d_k^{v}}{pb}}\right)\right] - \left[\sum_{m=0}^{M-1}\frac{2(x_{th}^{n+1}d_k^{v})^m}{m!\,\Gamma(a)b^a p^m}\times\left(\frac{x_{th}^{n+1}d_k^{v}\,b}{p}\right)^{\frac{a-m}{2}} K_{a-m}\left(2\sqrt{\frac{x_{th}^{n+1}d_k^{v}}{pb}}\right)\right]\right] \tag{16}$$

The above expression is obtained with the help of [21, eq. (8.352.1)] and [21, eq. (3.471.9)].

C. Bit Error Outage

The BEO for MAM scheme, $\mathrm{BEO}_k^{\mathrm{MAM}}$Is still the same as (14). For the FAM scheme as mentioned in (13) exploiting (15), we have:

$$BEO_k^{FAM} = \int_0^{x_{th}^1}\int_0^{\infty} f_{X_k|\bar{X}_k}(\xi|\bar{\chi})f_{\bar{X}_k}(\bar{\chi})\,d\bar{\chi}d\xi$$
$$= 1 - \sum_{m=0}^{M-1}\frac{2(x_{th}^{1}d_k^{v})^m}{m!\,\Gamma(a)b^a p^m}\left(\frac{x_{th}^{1}d_k^{v}\,b}{p}\right)^{\frac{a-m}{2}} \times K_{a-m}\left(2\sqrt{\frac{x_{th}^{1}d_k^{v}}{pb}}\right) \tag{17}$$

V. Numerical Results

In this section, numerical results are provided to investigate the performance of the proposed adaptive modulation scheme. For the simulation setup, we assume a cell radius $R = 1$ km with K uniformly distributed users; the reference distance is $r_h = 100$ m. The path-loss exponent v equals to 3.8, and the parameters of shadow fading are set as $a = 10$, and $b = \frac{1}{a}$ ($b = \frac{E[z_k]}{a}$, the expectation of z_k is usually chosen to be 1) [12]. However, without loss of generality, equal transmit power is assumed for all users.

Fig. 2 depicts the ASE theoretical results of the ZF and MRC receivers for different users' arrangements or distance vectors (DV). In fact, the various groups of users' distances are considered to show the impact of the users' arrangement on the ASE. The users' distances are normalized to the reference distance (r_h) when calculating the variance of each group of distances.

Fig. 2 shows that different variances of users' distances have different maximum ASE. The reason is that for each users' arrangement, we have different approximated SINR thresholds for each power. It shows that when the variance of users' distances for one arrangement increases, then the maximum ASE value in high power decreases. This is due to the near-far effect of all users' aggregate received power.

Fig.3 shows that the average spectral efficiency is increased by increasing the variance in the low power range. Therefore, it can be justified that we mitigate all the interferences in the low power transmission. Furthermore, by increasing the variance, close users to the BS will have better SINR values and, thus, higher quantities of spectral efficiency. Nevertheless, these near users will not interfere with the far users because of the interference mitigation.

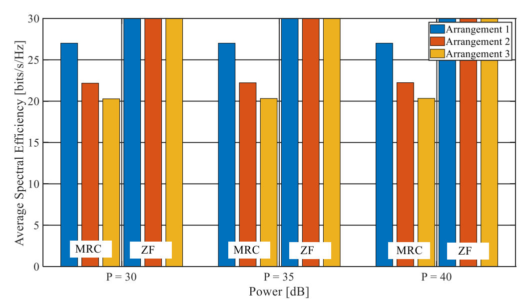

Fig. 2: ASE in the MAM scheme for different users' arrangements; Arrangement 1: var.(normalized to r_h) = 0.62, Arrangement 2: var. = 2.5, Arrangement 3: var. = 3.90; M = 2048, K = 5

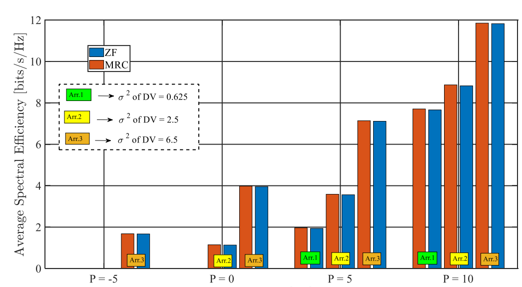

Fig. 3. ASE in the MAM scheme for different users' arrangements in low power approximation; M = 512, K = 5

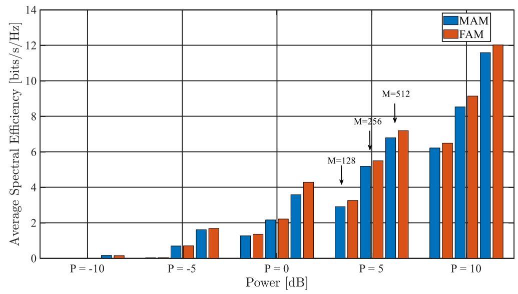

Fig. 4. Impact of M on the Average Spectral Efficiency in low power approximation, K =5

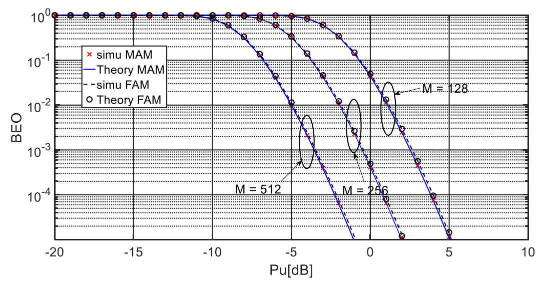

Fig. 5: Impact of M on the BEO with K = 5, for an arbitrary distance with MRC receiver

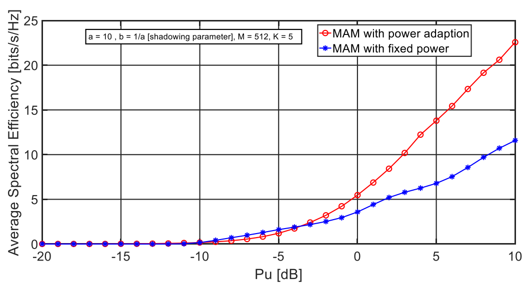

Fig. 6: ASE for MAM with joint power control, M = 512, K = 5

This interference mitigation capability is the same as ZF receivers which null out interferences by pseudo-inverse channel matrix.

In Fig. 4, we can see that higher average spectral efficiency can be achieved by increasing the number of BS antennas, and the two schemes of adaptation come close to each other.

Fig. 5 shows the BEO of the MAM and FAM schemes. It is observed that the analytical results are in perfect agreement with the simulation results at low power transmission. Furthermore, by increasing the number of BS antennas, both MAM and FAM show identical performance.

Fig. 6 shows the ASE of MAM with joint power control. As can be readily observed, MAM with joint power adaptation can significantly increase the achievable ASE.

VI. Conclusion

In this paper, a low-complexity adaptive scheme in an uplink multiuser transmission for a large number of antennas at the receiver was evaluated where receiver employees MRC to combine data streams. In this aim, theoretical closed-form expressions were calculated for both ASE and BEO. Results showed that the performance gap between MRC and ZF receivers is significantly high by increasing the transmit power to high values. We also observed that the maximum value of ASE greatly depends on the mean and variance of the users' distances. Moreover, in general cases and all power ranges, the expressions of PDF and bit error probability are so complicated to compute SINR thresholds. This motivated us to focus on low power transmission, an interesting range of power for MRC receivers. At low power approximation, increasing the variances of distance vectors play an excellent role where both the ZF and MRC receivers give a comparable performance. Moreover, increasing the number of antennas at BS can bear higher values for both ASE and BEO, and also the gap between the two schemes of adaptation would be diminished. Finally, we enhanced the ASE for our MAM scheme by using the optimal power allocation strategy and switching threshold.

Appendix (Proof Theorem)

This appendix evaluates the conditional PDF of SINR on $\bar{\chi}_k$ in Eq. (7).

Rewriting Eq. (5) for the channel we have:

$$\chi_k = \frac{\bar{\chi}_k}{M} \frac{X}{p \sum_{\substack{i=1 \\ i \neq k}}^{K} z_{ik} + 1} \tag{18}$$

$$X = \|\boldsymbol{h}_k\|^2, \; z_{ik} = \beta_i y_{ik} \tag{19}$$

As we know:

$$\beta_i \sim G(a, \lambda_i) \tag{20}$$

$$y_k = y_{ik} = \frac{\left|\mathbf{h}_k{}^H \mathbf{h}_i\right|^2}{\|\mathbf{h}_k\|^2} \sim \exp(1) \tag{21}$$

where omitting index i in (21) can be asymptotically true only for a small number of users (K). As we know:

$$\lambda_i = \frac{b}{d_i{}^v}, \; d_i{}^v = \left({}^{r_i}/{}_{r_h}\right)^v \tag{22}$$

By fixing β_i and writing $y_k = z_{ik}/\beta_i$, we obtain the conditional PDF $f_{z_{ik}|y_k}(z|y)$ as follows:

$$f_{z_{ik}|y_k}(z|y) = \frac{z^{a-1}}{\Gamma(a)(y\lambda_i)^a} e^{-\frac{z}{y\lambda_i}} \tag{23}$$

So, the distribution for the RV z_{ik} can be described as:

$$f_{z_{ik}}(z) = E_y\left[f_{z_{ik}|y_k}(z|y)\right] \tag{24}$$

Considering (23), each z_{ik} can be viewed as a Gamma-distributed RV with shape parameter a, and random scale parameter $y\lambda_i$. It follows that given y_k, the RV Z is the sum of $K-1$ independent gamma-distributed RVs with different second parameters. Finally, the PDF of Z conditioned on y_k can be computed by Satterthwaite Procedure. This procedure is described in Lemma 1.

Lemma 1: $\frac{v\,U}{E\{U\}}$ has an approximate χ^2 distribution (chi-square) with the following degree of freedom:

$$v = \frac{\left(\sum_{i=1}^{k} a_i E\{U_i\}\right)^2}{\sum_{i=1}^{k} (a_i E\{U_i\})^2 / v_i} \tag{25}$$

We can assume $X_i = a_i U_i \sim G(K_i, \Xi_i)$ where $K_i = v_i/2$ and $\Xi_i = 2a_i$.

$$X = \sum_i X_i \;\; ; \;\; \frac{\nu\, X}{E\{X\}} \sim \chi(\nu) \tag{26}$$

$$\nu = \frac{\left(\sum_{i=1}^{k} K_i \Xi_i\right)^2}{\sum_{i=1}^{k} (K_i \Xi_i)^2 / 2K_i} \;\; , E\{X\} = \sum_i K_i \Xi_i \tag{27}$$

Therefore, we have:

$$X \sim G(\,K_{sum}\,,\,\Xi_{sum}) \tag{28}$$

$$K_{sum} = \frac{\left(\sum_{i=1}^{k} K_i \Xi_i\right)^2}{\sum_{i=1}^{k} K_i {\Xi_i}^2} \;,\; \theta_{sum} = \frac{\sum_{i=1}^{k} K_i \Xi_i}{K_{sum}} \tag{29}$$

In our case, we substitute K_i $with\ a\ for\ all\ i,\ \Xi_i = y\lambda_i$ and also X_i with conditional z_{ik} on y_k so we have approximated conditional pdf for Z as bellow:

$$Z \sim G(\,\theta_k\,, y\Omega_k\,) \tag{30}$$

where:

$$\theta_k = a\frac{\left(\sum_{i=k,i\neq k}^{K} \lambda_i\right)^2}{\sum_{i=k,i\neq k}^{K} {\lambda_i}^2} \;, \Omega_k = a\frac{\sum_{i=k,i\neq k}^{K} \lambda_i}{\theta_k} \tag{31}$$

So, the unconditional distribution of Z can be obtained by averaging the PDF over y_k, that is:

$$f_Z(Z) = \int_0^\infty f_{Z|y}(Z|y) f_y(y)\, dy$$
$$= \frac{Z^{\theta_k - 1}}{\Gamma(\theta_k){\Omega_k}^{\theta_k}} \int_0^\infty y^{-\theta_k} e^{-\left(y + \frac{Z}{y\Omega_k}\right)} dy \tag{32}$$

Finally, we have:

$$f_{X_k|\bar{X}_k}(\chi|\bar{\chi}) \approx \frac{\left(\frac{M}{\bar{\chi}}\right)^M \chi^{M-1}\, exp\left(\frac{\bar{\chi}}{2Mp\Omega_k\chi} - \frac{M}{\bar{\chi}}\chi\right)}{(p\Omega_k)^{\frac{\theta_k}{2}}\Gamma(M)\Gamma(\theta_k)}$$
$$\times \left(\sum_{i=0}^{M} \binom{M}{i} \Gamma(\theta_k + i)\, \Gamma(i+1) \times \dagger_{i,k}(\chi, \bar{\chi})\right) \tag{33}$$

where:

$$\dagger_{i,k}(\chi, \bar{\chi})$$
$$= \left(\frac{M\chi}{\bar{\chi}}\right)^{-\frac{1}{2}(\theta_k + 2i)} W_{-\frac{1}{2}(\theta_k+2i)\,,\; \frac{1}{2}(\theta_k - 1)} \left(\frac{\bar{\chi}}{Mp\Omega_k\chi}\right) \tag{34}$$

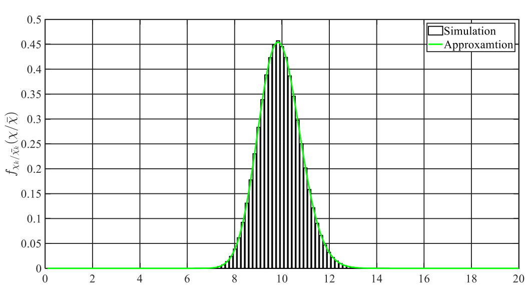

Fig. 7: $f_{X_k|\bar{X}_k}(\chi|\bar{\chi})$, M = 128 , K = 5 and $\bar{\chi} = 10, p = 0\ dB$

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