Reflection Design With LS Channel Estimation for RIS-enhanced OFDM Systems

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Abstract—**Reconfigurable intelligent surface (RIS) technology is a promising solution for improving the link quality of wireless communication systems. This study considers the problem of optimizing the passive reflections in a RIS-enhanced orthogonal frequency division multiplexing (OFDM) system. The problem is challenging since the channel state information in practical systems is seldom precisely known, and each reflection can only use a common phase shifter for all the subcarriers. Accordingly, the optimization procedure performed herein considers the effects of channel estimation errors, and seeks to optimize the reflections in accordance with the max-capacity criterion. Owing to the intractable non-convex nature of the problem, existing methods approximate the max-capacity design by maximizing the signal-to-noise ratio. By contrast, the present study adopts the gradient ascent update method to obtain the local optimum solution of each reflection directly. The simulation results confirm the superiority of the proposed reflection design and reveal that the max-SNR criterion is not a good approximation of the max-capacity design for RIS-assisted OFDM systems.**

Index Terms **–Reconfigurable intelligent surface (RIS); orthogonal frequency division multiplexing (OFDM); least square (LS); channel estimation; reflection optimization.**

I. INTRODUCTION

Reconfigurable intelligent surface (RIS) technology, also known as intelligent reflection surface (IRS), has emerged as a promising technique for improving the energy efficiency, link quality, and coverage of wireless communication systems. By deploying massive low-cost reflections, the RIS adjusts and reflects the phases of the transmitted signals in such a way as to concentrate the received signal power at the destination; thereby improving the signal-to-noise ratio (SNR). Notably, in addition to their low-cost and low-energy properties, RIS-assisted structures also make possible full-duplex transmission without self-interference [1]-[7].

RIS-assisted systems require accurate channel state information (CSI) to properly design the passive reflections. Thus, previous studies on passive reflection design generally assume perfect cascaded CSI [2], [4], [5]. However, in practical systems, such a condition is seldom achieved. Consequently, pilots are inserted in the transmitted signals in order to estimate the CSI at the receiver end [4].

Due to the lack of A/D and D/A converters at the RIS terminal, RIS-assisted systems can only utilize the time-domain resource to estimate the cascaded CSI of the access point-to-RIS-to-user equipment (AP-RIS-UE) channel [6]. Essentially, the greater the number of reflections used, the higher the number of time-domain pilots needed to estimate the CSI, and hence the lower the spectral efficiency.

The channel estimation (CE) problem has been widely studied in RIS-assisted wireless systems [4], [6]-[10]. However, very few studies have considered the reflection design problem for the practical case of imperfect CSI [1], [4], [7]. In [7], the authors jointly considered the reflection and channel estimation problems for mmWave MIMO systems. The authors in [1] developed a reflection design to maximize the capacity (max-capacity) of a RIS-assisted OFDM system with imperfect CSI. Since the associated optimization problem is not convex, and each reflection can only use a common phase shifter on all the subcarriers, the authors designed each phase shifter in such a way as to align the phase of the strongest multipath and increase the received signal power accordingly. In other words, the design in [1] is essentially an approximation of the maximum SNR (max-SNR) design rather than the max-capacity design.

The present study employs a gradient approach to conduct reflection optimization with max-SNR and max-capacity sub-optimally. For each criterion, the reflections are updated separately until convergence. For the reflection design based on the max-SNR criterion, an optimum combination strategy is proposed to further improve the SNR performance. The simulation results show that the proposed design outperforms existing designs in terms of both a higher SNR and a higher transmission rate. It is additionally shown that the max-SNR criterion is not a proper approximation of the max-capacity design for RIS-assisted OFDM systems.

The remainder of this paper is organized as follows. Section II describes the received signal and system model and introduces the least-square (LS) channel estimation strategy for RIS-assisted systems. Section III presents the proposed reflection optimization approach. Section IV presents and discuses the simulation results. Finally, Section V provides some brief concluding remarks.

II. SIGNAL MODEL AND CHANNEL ESTIMATION

A. System Model

The present study considers an uplink RIS-assisted OFDM system, in which a RIS is deployed at an appropriate location to help the UE relay its signals to the AP, as shown in Fig. 1. At the UE side, the kth OFDM symbol $\mathbf{x}_k := \begin{bmatrix} X_{k,0} & X_{k,1} & \cdots & X_{k,N-1} \end{bmatrix}^T$ is transferred by an N-point inverse discrete Fourier transform (IDFT) matrix, and a cyclic prefix (CP) of length L_{CP} is then added to the front end. (Herein, the length of the CP is assumed to be greater than the maximum delay spread of the cascaded channels.)

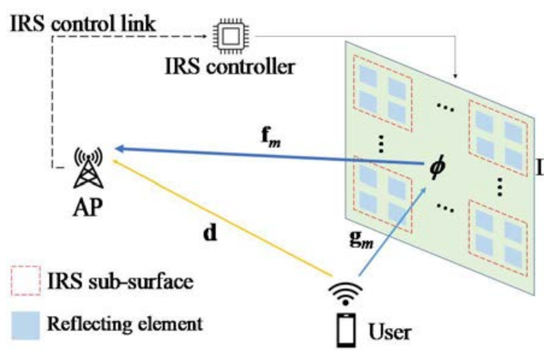

Fig 1. Uplink RIS-assisted OFDM system.

At the AP end, the CP of the received signal is removed, and the DFT matrix is applied. The resulting equivalent baseband signal of the kth received OFDM signal in the frequency domain is obtained as

$$\mathbf{y}_k = \mathbf{X}_k\left(\mathbf{d}+\sum_{m=1}^{M}\mathbf{g}_m\phi_m \odot \mathbf{f}_m\right)+\mathbf{w}_k, \tag{1}$$

where $\mathbf{y}_k = \begin{bmatrix} y_{k,0} & y_{k,1} & \cdots & y_{k,N-1}\end{bmatrix}^T \in \mathbb{C}^{N\times 1}$ is the kth received OFDM symbol; $\mathbf{X}_k = \mathrm{Diag}\{\mathbf{x}_k\}$ makes the vector $\mathbf{x}_k$ a diagonal matrix; $\mathbf{d}\in\mathbb{C}^{N\times 1}$ is the channel frequency response (CFR) of the UE-to-AP channel; $\mathbf{g}_m \in \mathbb{C}^{N\times 1}$ is the CFR of the UE-to-mth RIS channel; $\phi_m = \beta_m e^{j\theta_m}$, where $\beta_m \in [0,1]$ and $\theta_m \in [0,2\pi)$, is the mth reflection coefficient; $\mathbf{f}_m$ is the CFR of the mth RIS-to-AP channel; and $\mathbf{w}_k = \begin{bmatrix} w_{k,0} & \cdots & w_{k,N-1}\end{bmatrix}^T \sim \mathcal{CN}\left(0,\sigma_n^2\mathbf{I}_N\right)$ is the additive white Gaussian noise (AWGN) vector.

By defining the equivalent cascaded CFR as $\mathbf{h}_m = \mathbf{g}_m \odot \mathbf{f}_m \in \mathbb{C}^{N\times 1}$, (1) can be rewritten in the following vector form:

$$\mathbf{y}_k = \mathbf{X}_k\left(\mathbf{d}+\mathbf{H}\phi\right)+\mathbf{w}_k, \tag{2}$$

where $\mathbf{H} = \begin{bmatrix}\mathbf{h}_1 & \cdots & \mathbf{h}_m\end{bmatrix} \in \mathbb{C}^{N\times M}$ represents all the reflecting channels and $\phi = \begin{bmatrix}\phi_1 & \cdots & \phi_M\end{bmatrix}^T$ is the phase shifter vector.

In designing the common phase shifter ϕ, it is necessary to estimate the channels of $\mathbf{H}$ and $\mathbf{d}$. Conventionally, this is achieved using the pilots at the transmitter end to detect the CSI. However, such an approach is inapplicable in the present case since only the composite channel $(\mathbf{d}+\mathbf{H}\phi)$ is to be estimated. The authors in [1] thus developed a transmission protocol for estimating the combined CSI of $\mathbf{H}$ and $\mathbf{d}$ by using the RIS reflection pattern ϕ as pilots. The following section restates the least-square (LS) channel estimation problem for RIS-assisted OFDM systems accordingly.

B. Channel Estimation

Let each transmission frame contain N_T OFDM symbols. Furthermore, assume that each frame consists of two sub-frames, where the first sub-frame contains T consecutive pilot symbols (wherein the pilots at both the transmitter end and the RIS end are used), and the second sub-frame is used for data transmission. Let $\mathbf{p}_i = \begin{bmatrix} p_1^{(i)} & \cdots & p_M^{(i)}\end{bmatrix}^T$ denote the pilots of the RIS reflection phase during the ith transmission of the training stage and set $\mathbf{X}_i = \mathbf{I}_N$. The received signal at the destination of the ith transmission can then be expressed as

$$\mathbf{y}_i = \begin{bmatrix}\mathbf{d} & \mathbf{H}\end{bmatrix}\begin{bmatrix}1\\ \mathbf{p}_i\end{bmatrix} + \mathbf{w}_i. \tag{3}$$

Collecting the T received pilot symbols as a matrix yields

$$\begin{aligned}\mathbf{Y} &= \begin{bmatrix}\mathbf{y}_1 & \cdots & \mathbf{y}_T\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{d} & \mathbf{H}\end{bmatrix}\begin{bmatrix}1 & \cdots & 1\\ \mathbf{p}_1 & \cdots & \mathbf{p}_T\end{bmatrix} + \begin{bmatrix}\mathbf{w}_1 & \cdots & \mathbf{w}_T\end{bmatrix}\\ &= \mathbf{F}_L\begin{bmatrix}\mathbf{d}_t & \mathbf{H}_t\end{bmatrix}\Phi + \mathbf{W}\end{aligned} \tag{4}$$

where

$$\Phi = \begin{bmatrix}1 & \cdots & 1\\ \mathbf{p}_1 & \cdots & \mathbf{p}_T\end{bmatrix} \tag{5}$$

and $\mathbf{d}_t$ and $\mathbf{H}_t = \begin{bmatrix}\mathbf{h}_{t,1} & \mathbf{h}_{t,2} & \cdots & \mathbf{h}_{t,M}\end{bmatrix}$ are the channel impulse responses (CIRs) of the direct and AP-RIS-UE channels, respectively. $\mathbf{F}_L$ collects the first L columns of the NxN DFT matrix. In addition, $\mathbf{W} = \begin{bmatrix}\mathbf{w}_1 & \cdots & \mathbf{w}_T\end{bmatrix}$ is the received noise matrix. The channels can then be estimated by the LS criterion with the pilots Φ as

$$\begin{bmatrix}\hat{\mathbf{d}} & \hat{\mathbf{H}}\end{bmatrix} = \mathbf{Y}\Phi^H\left(\Phi\Phi^H\right)^{-1} = \begin{bmatrix}\mathbf{d} & \mathbf{H}\end{bmatrix} + \mathbf{N}, \tag{6}$$

where $\mathbf{N} = \mathbf{W}\Phi^H(\Phi\Phi^H)^{-1} \in \mathbb{C}^{N\times(M+1)}$ is the channel estimation error resulting from LS estimation. It is noted that T is set as $T \geq M+1$ to provide sufficient degrees of freedom for the estimation process. Moreover, as suggested in [1], $\Phi = \mathbf{F}_T(1:M+1,:)$ is set to achieve the minimum LS value, and $\mathbf{N} \sim \mathcal{CN}\left(0,\sigma_n^2(M+1)\mathbf{I}_N\right)$ accordingly. Herein, $\mathbf{F}_T(1:M+1,:)$ collects the first (M+1) rows of the TxT DFT matrix.

III. Optimum Reflection Design

This section describes the reflection optimization problem considered in the present study based on the max-capacity criterion. The section commences by deriving the transmission rate as a function of the estimated CSI. The associated optimization process is then formally derived.

A. Transmission Rate

The received signal over the LS-estimated channel can be written as

$$\begin{aligned}\mathbf{y}_k &= \mathbf{X}_k\left(\mathbf{d}+\mathbf{H}\phi\right)+\mathbf{w}_k, \\ &=\mathbf{X}_k\left(\left(\left[\hat{\mathbf{d}} \quad \widehat{\mathbf{H}}\right]+\mathbf{N}\right)\begin{bmatrix}1\\ \phi\end{bmatrix}\right)+\mathbf{w}_k \\ &= \mathbf{X}_k\left[\hat{\mathbf{d}} \quad \widehat{\mathbf{H}}\right]\begin{bmatrix}1\\ \phi\end{bmatrix}+\mathbf{E}_k\end{aligned} \tag{7}$$

where

$$\mathbf{E}_k=\mathbf{X}_k\mathbf{N}\begin{bmatrix}1\\ \phi\end{bmatrix}+\mathbf{w}_k \tag{8}$$

and $\mathbf{E}_k \sim \mathcal{CN}\left(0,\sigma_n^2\left(\sigma_s^2\left(M+1\right)+1\right)\mathbf{I}_N\right)$ denotes the equivalent noise contributed from the channel estimation error and the observation noise. Considering the transmission rates of all the subcarriers, the length of the pilots, and the length of the CP, respectively, and dropping the time index *k*, the overall transmission rate can be expressed as

$$C\left(\phi\right)=\frac{N_T-\left(M+1\right)}{N_T}\frac{N}{N+L_{CP}}\sum_{n=0}^{N-1}\log\left(1+\frac{\sigma_s^2\left|\mathbf{c}_n\left(\phi\right)\right|^2}{\sigma_n^2\left(\sigma_s^2\left(M+1\right)+1\right)}\right) \tag{9}$$

where the combined channel gain is given by

$$\mathbf{c}=\left[\hat{\mathbf{d}} \quad \widehat{\mathbf{H}}\right]\begin{bmatrix}1\\ \phi\end{bmatrix}=\mathbf{F}_L\left[\hat{\mathbf{d}}_t \quad \widehat{\mathbf{H}}_t\right]\begin{bmatrix}1\\ \phi\end{bmatrix} \tag{10}$$

and N_T is the total number of OFDM symbols in a frame.

B. Optimum Reflection Design Based on Max-Capacity Criterion

Considering (9), the optimum reflection design with max-capacity can be formulated as

$$\begin{aligned}&\max_{\{\theta_i\}}\sum_{n=0}^{N-1}\log\left(1+\frac{\sigma_s^2\left|\mathbf{c}_n\left(\phi\right)\right|^2}{\sigma_n^2\left(\sigma_s^2\left(M+1\right)+1\right)}\right)\\ &s.t.\\ &\phi_i=e^{j\theta_i},\quad i=1,\cdots,M.\end{aligned} \tag{11}$$

The problem in (11) is not convex [11]. As a result, computing the optimum solution directly is infeasible. In [1], the problem was thus approximated by the max-SNR problem, i.e.,

$$\begin{aligned}&\max_{\{\theta_i\}}\sum_{n=0}^{N-1}\left|\mathbf{c}_n\left(\phi\right)\right|^2=\max_{\{\theta_i\}}\left\|\left[\hat{\mathbf{d}}_t \quad \widehat{\mathbf{H}}_t\right]\begin{bmatrix}1\\ \phi\end{bmatrix}\right\|_2^2\\ &s.t.\\ &\phi_i=e^{j\theta_i},\quad i=1,\cdots,M.\end{aligned} \tag{12}$$

However, the problem in (12) is still not convex. Accordingly, the authors in [1] provided a suboptimum solution for each reflection phase shift that aligns the phase of the strongest CIR as

$$\theta_i=-\angle h_{t,i,\hat{l}}+\angle d_{t,i,\hat{l}},\quad i=1,\cdots,M, \tag{13}$$

where $h_{t,i,\hat{l}}$ and $d_{t,i,\hat{l}}$ are the strongest paths of $\mathbf{h}_{t,i}$ and $\mathbf{d}_{t,i}$, respectively. By contrast, the present study adopts the gradient ascent approach to further improve the suboptimal solution of (12). In particular, defining $\theta=\left[\theta_1 \quad \cdots \quad \theta_M\right]^T$ and letting $\hat{\theta}^{(n)}$ be the estimation of θ in the *n*th iteration, the reflection phases obtained from (12) are updated as follows:

$$\hat{\theta}^{(n+1)}=\hat{\theta}^{(n)}+\alpha\nabla_\theta J_1, \tag{14}$$

where α is the step size, and $\nabla_\theta J_1$ is the gradient of the objective of (12), which can be computed as

$$\nabla J_1=2\,\mathrm{Im}\left\{\phi^*\odot\mathbf{M}_{21}\right\}+2\,\mathrm{Im}\left\{\phi^*\odot\left(\mathbf{M}_{22}\phi\right)\right\} \tag{15}$$

in which $\mathbf{M}_{21}=\widehat{\mathbf{H}}_t^H\hat{\mathbf{d}}_t$ and $\mathbf{M}_{22}=\widehat{\mathbf{H}}_t^H\widehat{\mathbf{H}}_t$ .Let $\theta^{(\infty)}=[\theta_1^{(\infty)},\cdots,\theta_M^{(\infty)}]^T$ and $\phi^{(\infty)}=[e^{j\theta_1^{(\infty)}},\cdots,e^{j\theta_M^{(\infty)}}]^T$ be the converged solution. In general, the performance of the gradient ascent method is sensitive to the initial value. Accordingly, in the present study, the initial value is chosen using the low-complexity strongest channel impulse response method proposed in [1] and given by (13). Once $\phi^{(\infty)}$ is obtained, an optimal phase rotation is then applied to further improve the performance. Toward this end, a new notation $\mathbf{u}=\widehat{\mathbf{H}}_t\phi^{(\infty)}$ is introduced, and the phase obtained from (13) is further optimized as

$$\begin{aligned}&P3:\max_{a_0}\left\|\left[\hat{\mathbf{d}}_t \quad \mathbf{u}\right]\begin{bmatrix}1\\ a\end{bmatrix}\right\|^2\\ &s.t.\quad \left|a\right|=1.\end{aligned} \tag{16}$$

The optimum solution of (16) is obtained as $a_{opt}=\exp\left(j\angle\mathbf{u}^H\hat{\mathbf{d}}_t\right)$, and stems from the following equivalence:

$$\left\|\hat{\mathbf{d}}_t+a\mathbf{u}\right\|^2=\left\|\hat{\mathbf{d}}_t\right\|^2+\left\|\mathbf{u}\right\|^2+a^*\mathbf{u}^H\hat{\mathbf{d}}_t+a\hat{\mathbf{d}}_t^H\mathbf{u}. \tag{17}$$

Finally, the common phase shifter is realized by

$$\phi_{opt} = a_{opt}\phi^{(\infty)}. \quad (18)$$

Alternatively, the gradient approach method can be applied to (11) directly. Through such an approach, the optimum reflection phases of (11) based on the maximum capacity criterion are obtained directly as

$$\hat{\theta}^{(n+1)} = \hat{\theta}^{(n)} + \alpha\nabla_{\theta}J_2, \quad (19)$$

where

$$\nabla_{\theta}J_2 = \sum_{n=0}^{N-1} 2\left(\sigma_E^2 + \left|\mathbf{c}_n\right|^2\right)^{-1} \times \mathrm{Im}\left\{e^{-j\theta} \odot \left(\begin{bmatrix}1 & \phi^H\end{bmatrix}\begin{bmatrix}\hat{\mathbf{d}}_t^H \\ \hat{\mathbf{H}}_t^H\end{bmatrix}\mathbf{F}_L^H\mathbf{e}_n\hat{\mathbf{H}}_t^H\mathbf{F}_L^H\mathbf{e}_n\right)\right\}, \quad (20)$$

in which $\mathbf{e}_n$ is a column vector with the nth element equal to one and all the other elements equal to zero, and $\sigma_E^2 = \sigma_n^2(\sigma_s^2(M+1)+1)$. For both (14) and (19), the optimization procedure is repeated iteratively until convergence.

IV. Numerical Results and Discussion

Assume that $M = 16$ relections are used, the total number of OFDM symbols of a frame is $N_T = 80$, where $T = 17$ is the number of training OFDM symbols ($T \geq M+1$), $N = 64$ for each OFDM block, and $L_{cp} = 16$. Let the DFT matrix be applied to the pilot sequence and assume that the AP-RIS, RIS-UE and AP-UE channels all have a power of one. Finally, let the SNR be defined as $SNR = 1/\sigma_n^2$. Fig. 2 shows the signal-to-interference plus noise ratio (SINR) performance of the reflection design in [1] and the proposed max-SNR design in (14). As can be seen, the proposed design outperforms that of [1] since it additionally employs the gradient update approach in (14) and phase combining strategy in (18) to improve the local solution. However, as shown in Fig. 3, neither max-SNR design maximizes the transmission rate. Notably, the two methods have an almost identical transmission performance, and hence it is inferred that the max-SNR criterion is not a good approximation of the max-capacity criterion in RIS-assisted OFDM systems. As expected, the proposed max-capacity design in (19) achieves a higher transmission rate than either of the max-SNR designs due to its use of the max-capacity criterion.

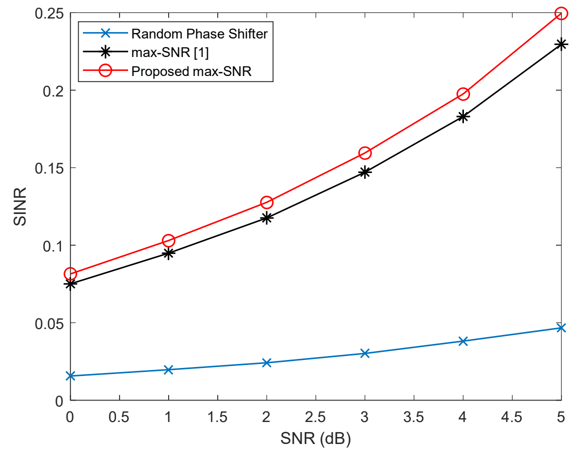

Fig. 2 SINR performance of [1] and proposed method in (14).

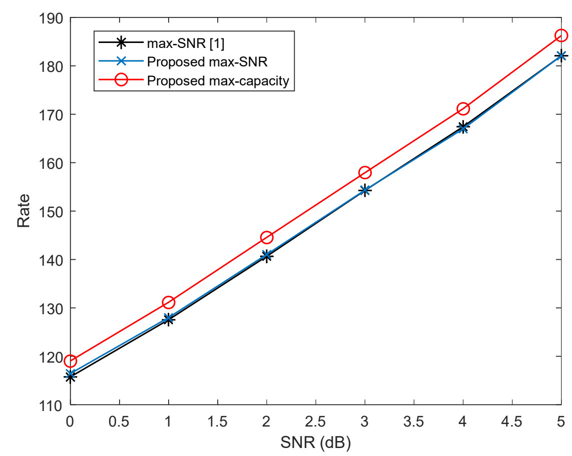

Fig. 3 Transmission rate performance of [1] and proposed methods in (14) and (19).

V. Conclusions

A novel reflection design with LS channel estimation has been proposed for RIS-assisted OFDM systems. The transmission rate has first been derived as a function of the channel estimation error and reflections of the RIS. The reflections have then been optimized based on a max-capacity criterion. Since the associated optimization problem is not convex, a gradient ascent method has been employed to determine the local optimum solution. The simulation results have confirmed the validity of the proposed design and have shown that the max-SNR criterion does not provide a proper approximation of the max-capacity criterion when designing the reflections of RIS-assisted OFDM systems since each reflection can only share the same phase shifter for all of the subcarriers.

VI. Acknowledgement

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