Performance improvement for windowed OFDM using pre-coding and sub-carriers interleaving

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Abstract— This paper discusses the out-of-band emission power reduction technique by window functions and performance improvement scheme for OFDM (Orthogonal Frequency Multiple Access) systems. The windowed OFDM is effective to suppress the out-of-emission band, however, the communication performance is degraded, comparing with the conventional OFDM system. To improve the performance in frequency selective fading channel, the pre-coding using the unitary transform is adopted. Moreover, the performance improvement scheme that data sequence between the unitary transform and IFFT/FFT is interleaved for the pre-coded windowed OFDM system is proposed. The BER (Bit Error Rate) is evaluated to show the effectiveness of the proposed scheme, comparing to the conventional pre-coded windowed schemes and the out-of-band emission power is also reduced.

Keywords— Windowed OFDM, Interleaving, Walsh-Hadamard Transforms, Pre-coding, Out-of-band emission reduction.

I. INTRODUCTION

Enhancing spectrum efficiency is an important issue for wireless systems because they are required higher data rate and higher traffic capacity. To suppress the out-of-band emission is also significant since the guard band among wireless systems can be narrower, and it is possible to improve spectrum usage. Filtered OFDM, FBMC (Filter Bank Multi-Carrier) and GFDM (Generalized Frequency Division Multiplex) had been proposed as modulation schemes to reduce out-of-band power for mobile systems [1][2]. The time domain windowing scheme that non-contiguous signal points in the symbol are compensated and windowed in time domain also had been proposed to suppress it for IEEE 802.11af based WLAN systems[3].

For flexible spectrum usage, the wideband non-contiguous OFDM and the technique to make spectrum notch in the OFDM spectrum had been proposed for dynamic spectrum access [4][5]. Both techniques use the window functions to reduce the side-lobe of the OFDM spectrum. If the side-lobe of the spectrum is not lower sufficiently, it has a potential to interfere with the other systems. Therefore, designing shapes of the window function is significant. However, communication performances of the windowed OFDM are worse than the conventional OFDM system, because the effective symbol is warped by the window functions. Generally, the function which has higher efficiency for suppressing the side-lobe, the BER performances are degraded more. Therefore, it is important to realize both better communication performance and higher capability for reducing out-of-band emission.

We had considered the performance improvement scheme for the windowed OFDM system. The guard interval extension scheme was considered to obtain better communication performance [6][7]. To improve the performance against multipath, the pre-coded OFDM had been propose. The scheme can be spread the influence of the frequency selective fading [8][9].

In this paper, the performance improvement scheme using pre-coded windowed OFDM is proposed. To suppress the out-of-band emission and improve communication performance under multipath environment, windowing and pre-coding by the unitary transform are used for the OFDM system. Moreover, the data sequence between the unitary transform and IFFT/FFT is pseudo-randomized as interleaving. The performance can be improved compared to the ordinary windowed and pre-coded windowed OFDM. And the out-of-band emission also can be reduced than the conventional OFDM system.

II. PRE-CODED WINDOWED OFDM

In this section, the transmission and reception scheme for the proposed windowed OFDM is explained. The transmitter and receiver structure are shown in Figure 1. The structure is based on the conventional OFDM transmitter and receiver [10].

The transmitted data is modulated by the primary modulation like QPSK, QAM and so on. The modulated data is performed Walsh-Hadamard (WH) transform to be robust for the influence of multipath [8][9].

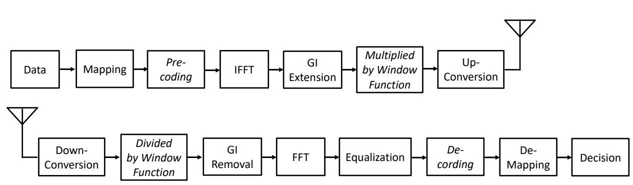


Figure 1 Transmitter-receiver structure of pre-coded windowed OFDM

$$d_{wh}(n) = \sum_{m=0}^{M-1} u(n,m) d(m) \quad (1)$$

where $d(m)$ is modulated data after mapping, $u(n, m)$ is Walsh matrix. $d_{wh}(n)$ is modulated data after WH transform. It is processed IFFT (IDFT) as follows

$$D_{wh}(k) = \frac{1}{M} \sum_{n=0}^{M-1} d_{wh}(n) \exp(j \frac{2\pi kn}{M}) \quad (2)$$

The guard interval signal is added to the symbol as follows.

$$D_{gi} = [D_{wh}(M - N_{gi}), \cdots, D_{wh}(M-1), D_{wh}(0), \cdots, D_{wh}(M-1)] \quad (3)$$

Where D_{gi} is a symbol after added the guard interval and N_{gi} is the number of samples for guard interval. The extended signal is multiplied by the window function $w(k)$ to suppress the side-lobe of the sub-carriers.

$$D_{win}(k) = D_{gi}(k) w(k) \quad (4)$$

After that, the symbol is up-converted to the radio frequency for transmission.

In this paper, Gaussian window function and Hamming window function are considered because the amplitude of those functions is not achieved to zero. Therefore, the BER performance becomes better than root raised cosine or Hanning window function that amplitude of those functions achieved zero at both ends [6][7].

Gaussian Window Function

$$w_{gus}(k) = \begin{cases} \exp\left[-\frac{1}{2} \left\{ \frac{\alpha (k - N/2)}{N/2} \right\}^2 \right] & (0 \le k \le N-1) \\ 0 & (k < 0, k > N-1) \end{cases} \quad (5)$$

where α denotes a coefficient the width of the window function and N is the length of the symbol including the GI.

Hamming Window Function

$$w_{han}(k) = \begin{cases} 0.54 - 0.46 \cos\left[\frac{2\pi k}{N} \right] & (0 \le k \le N-1) \\ 0 & (k < 0, k > N-1) \end{cases} \quad (6)$$

The shapes of the window functions are shown in Figure 2.

In the receiver side, the received signal is down-converted and sampled. The signal is divided by the window function to reconstruct the symbol as follows;

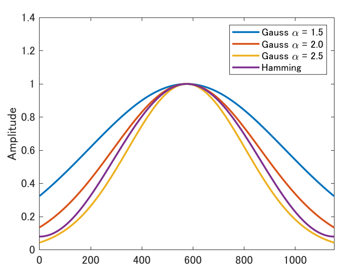


Figure 2 Gaussian and Hamming Window Function (N=1152)

$$R_{win}(k) = \frac{R_{dc}(k)}{w(k)} \quad (7)$$

where $R_{dc}(k)$ denotes the down-converted signal to the baseband. The signal is removed GI and processed FFT (DFT).

$$R_{gi} = [R_{win}(N_{gi}), \cdots, R_{win}(N-1)] \quad (8)$$

$$r_{ft}(n) = \sum_{k=0}^{M-1} R_{gi}(k) \exp(-j \frac{2\pi kn}{M}) \quad (9)$$

where $R_{gi}(k)$ is the received signal after removed GI. The signal is equalized. In this consideration, zero-forcing equalization is used as follows;

$$r_{eq}(n) = \frac{r_{ft}(n)}{H(n)} \quad (10)$$

where $H(n)$ is a transfer function of the channel. The equalized signal is processed by unitary transform using Walsh matrix to demodulate as follows.

$$d_r(n) = \sum_{m=0}^{M-1} u(n,m) r_{eq}(m) \quad (11)$$

III. Performance Evaluation of Windowed OFDM

In this section, BER performances of the pre-coded windowed OFDM system. The parameters of the OFDM signal are shown in Table 1. They are the same as the parameter of the LTE system [11]. The channel in time domain is expressed following equation.

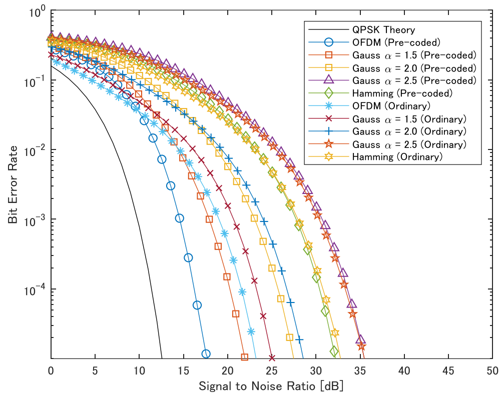
$$h(t) = \sum_{n=0}^{N-1} A_n \delta(t - \tau_n) \quad (12)$$

where $\delta(t)$ denotes delta function to express the impulse, A_n and τ_n are amplitude and delay time of the path, respectively.

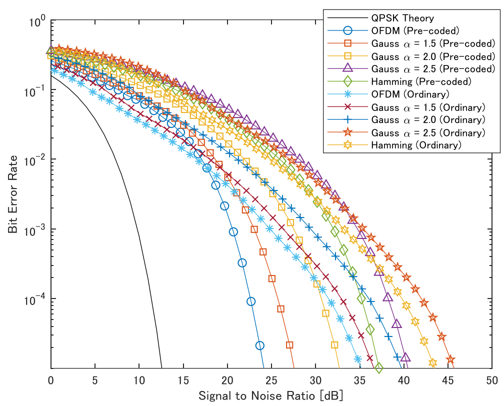
In Figure 3, the BER (Bit Error Rate) performances are shown when the channel is Extended Pedestrian A (EPA) model and two-path model. The delay time and power of the channels are shown in Table 2 [12]. The BER performances of pre-coded windowed OFDM are improved to obtain lower BER less than 10^{-2} in the most of the cases. The pre-coding is more effective when the power of multipath signals is larger and the α of the Gaussian window function is smaller. Using Hamming window function, the performance is also improved in EPA channel. When Gaussian window function with α = 2.5 is used in the two-path channel, the performance is worse than the conventional windowed OFDM.

Table 1 OFDM parameters

Parameters	Value
Number of Sub-carriers	1024
Sub-carrier Interval	15 [kHz]
Effective Symbol Duration	66.7 [us]
Guard Interval Duration	8.33 [us]
Primary Modulation	QPSK



(a) two-path channel



(b) EPA channel

Figure 3 BER performance of pre-coded windowed OFDM

In Figure 4, the BERs of each sub-carrier of the OFDM and windowed OFDM system are shown. In the conventional OFDM and windowed OFDM system, the BERs are varied depending on the channel in the frequency domain. It is the influence of the frequency selective fading. To adopt pre-coding, the influence of the frequency selective fading is spread into the all sub-carriers and BER of each sub-carriers is almost the same. The averaged BER performance of all sub-carriers becomes better than the non-pre-coding system. However, in the pre-coded windowed OFDM system, the BERs are different depending on each sub-carrier. Especially, BER becomes higher when the sub-carrier number is smaller.

Utilizing the characteristics, the important data can be transmitted by the sub-carrier which obtains better BER. However, the averaged BER of all sub-carriers is degraded. In the next section, the scheme to mitigate the difference of the BERs in each sub-carrier and to obtain a better averaged BER is proposed.

IV. Improvement for Pre-coded Windowed OFDM

The influence of the window function in the receiver side is considered. The received signal after down-conversion is expressed as equation (13). The influence of the multipath is spread by all sub-carriers in the pre-coded OFDM scheme, therefore the consideration are shown under AWGN environment in this section.

$$R_{dc}(k) = D_{win}(k) + Z(k) \tag{13}$$

where $Z(k)$ denotes noise. The signal is divided by the window function as well as equation (7).

$$R_{win}(k) = \frac{D_{win}(k)}{w(k)} + \frac{Z(k)}{w(k)} \tag{14}$$

The first term is reformed to the same as the pre-coded OFDM symbol. I focused on the noise component in the second term of equation (14), which is different from the conventional pre-coded OFDM system.

In this consideration, it is assumed that the guard interval is sufficiently shorter than the effective symbol duration. The symbol is processed DFT(FFT) as follows,

$$\begin{aligned} z_{ft}(n) &= \sum_{k=0}^{M-1} \frac{Z(k)}{w(k)} \exp(-j\frac{2\pi kn}{M}) \\ &= \left\{ \frac{1}{M} \sum_{k=0}^{M-1} \frac{1}{w(k)} \exp(-j\frac{2\pi kn}{M}) \right\} * z(n) \\ &= \sum_{k=0}^{M-1} W_{inv}(n-k) z(k) \end{aligned} \tag{15}$$

where $z(n)$ is the noise in time domain, W_{inv}(n) is the signal processed after FFT from $1/w(k)$ and * denotes the convolution. Under AWGN environment, $H(n)$ can be ignored for equalization. The $z_{ft}(n)$ is processed by the unitary transform using Walsh matrix from equation(11).

$$\begin{aligned} z_r(n) &= \sum_{m=0}^{M-1} u(n,m) \sum_{k=0}^{M-1} W_{inv}(n-k) z(k) \\ &= \sum_{k=0}^{M-1} z(k) \sum_{m=0}^{M-1} u(n,m) W(m-k) \end{aligned} \tag{16}$$

$$W_u(n) = \sum_{m=0}^{M-1} u(n,m) W(m-k) \tag{17}$$

The Walsh matrix is contained in cyclic sequences. For example, 8×8 Walsh matrix is shown as follows:

$$u(8,8) = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \end{bmatrix} \tag{17}$$

In lower columns of the matrix, the codes are continuous and periodical. Therefore, the averaged noise power becomes different among the sub-carriers after the unitary transform. The averaged noise power is shown in Figure 5(a). When the neighbor codes are the same, the correlation with the noise then becomes higher, and the power of the noise also becomes higher. The tendency is the same in both Gaussian window function and Hamming window function.

Table 2 Delay time and power of channels

	EPA Model	2 path model
Excess tap delay: t_n[ns]	0, 30, 70, 90, 110, 190, 410	0, 1000
Relative power: A_n [dB]	0.0, -1.0, -2.0, -3.0, -8.0, -17.2, -20.8	0.0, -3.0

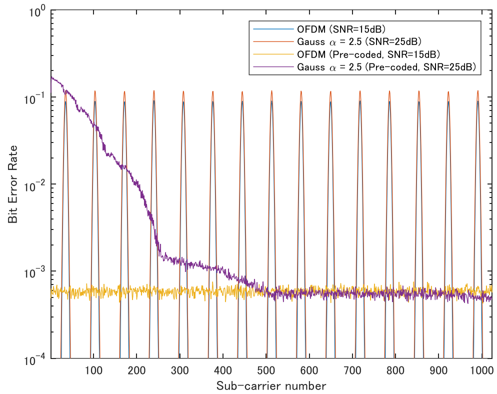
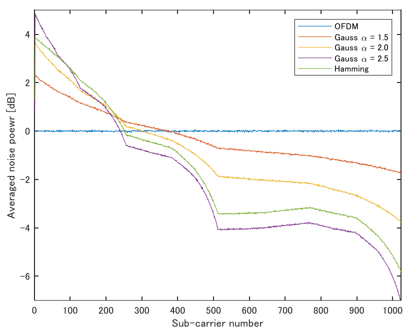
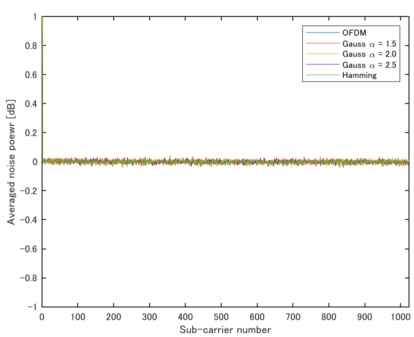


Figure 4 BER performance of each sub-carrier (two-path channel)



(a) Pre-coded windowed OFDM



(b) Pre-coded windowed OFDM with interleaving

Figure 5 Averaged noise power after WH transform

To improve the BER performance, the variance of the $W_u(n)$ should be lower. I propose the order of the received signal before the unitary transform after DFT(FFT) is exchanged randomly. The proposed transmitter-receiver structure is shown in Figure 6. $r_{eq}(n)$ is randomized as follows:

$$r_{il}(n) = r_{eq}(n_i) \tag{18}$$

where n_i denotes a pseudo-random integer from the substitution table. To correct the order, $d_{wh}(n)$ is also interleaved in the transmitter side.

$$d_{il}(n_i) = d_{wh}(n) \tag{19}$$

The $W_u(n)$ becomes as follow:

$$W_u(n) = \sum_{m=0}^{M-1} u(n,m) W(n_i - k) \tag{20}$$

In Figure 7, the averaged variance of the $W_u(n)$ using Gaussian window function is shown. The order of the $W(n_i)$ is exchanged at random by the interleaving. The variance becomes lower comparing with the ordinary system to adopt interleaving. Moreover, the power of the noise in each sub-carrier after unitary transform $z_{ft}(n)$ becomes almost constant as in Figure 5(b). Therefore, the average BER performance of all sub-carriers will be improved.

V. Performance Evaluation of Proposed System

In this section, performances are evaluated to show the effectiveness of the proposed scheme described in the previous section. Figure 8 shows the BER performance of the proposed pre-coded windowed OFDM system. The parameters of the OFDM and channel environment are the same as Section III. The BER performance can be improved to use the proposed scheme comparing the conventional pre-coded and non-pre-coded windowed OFDM systems. The BER of proposed pre-coded Gaussian windowed OFDM system α = 2.5 becomes about 4.5dB better than the conventional pre-coded windowed OFDM to obtain BER=10^{-4}. When α equals 1.5, the effect becomes less and the BER is improved about 1dB. In Figure 9, the BER performance in each sub-carrier is shown. It is confirmed that the difference of the BER can be mitigated by using the proposed scheme, therefore averaged BER of all sub-carrier can be improved.

In Figure 10, the spectrum of the proposed system and conventional OFDM are shown. It is confirmed that the Gaussian windowed OFDM α = 2.5 and Hamming windowed OFDM can be reduced the out-of-emission power about 20dB.

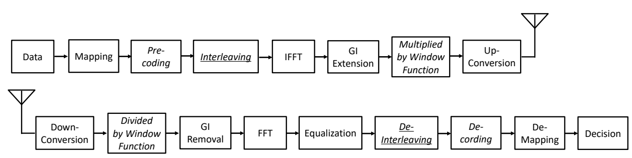


Figure 6 Modified transmitter-receiver structure of pre-coded windowed OFDM

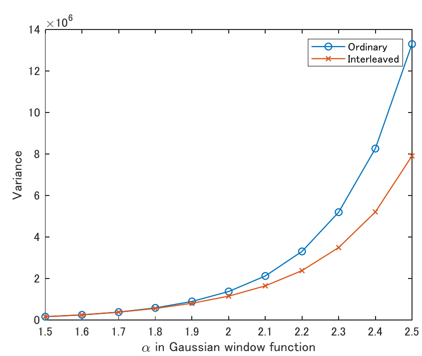
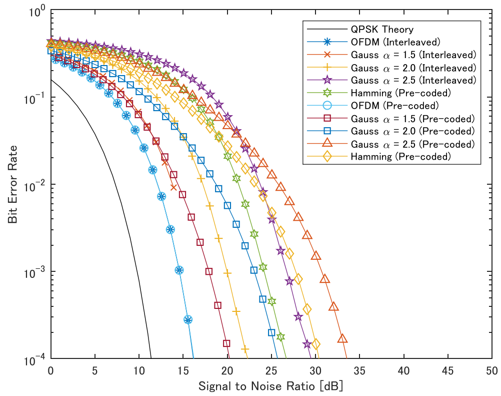
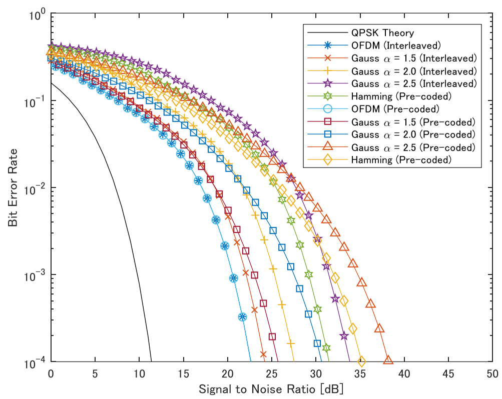


Figure 7 Variance of the $W_u(n)$ with Gaussian window



(a) two-path channel



(b) EPA channel

Figure 8 BER performance of pre-coded windowed OFDM

VI. Conclusion

In this paper, the pre-coded windowed OFDM is proposed to suppress the out-of-band emission and improve the performance in frequency selective fading channel. In the proposed scheme, the interleaving is adopted between the pre-coding and IFFT/FFT procedures to mitigate the power difference of noise among the sub-carriers. The BER performance can be improved and the out-of-band emission power can be reduced by the proposed pre-coded windowed OFDM system. Therefore, it is effective for both out-of-emission band suppression compared to the OFDM system and performance improvement compared with the conventional OFDM system.

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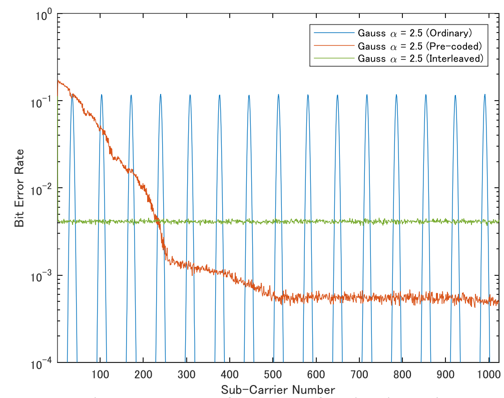

Figure 9 BER performance of each sub-carrier (two-path channel, SNR = 25dB)

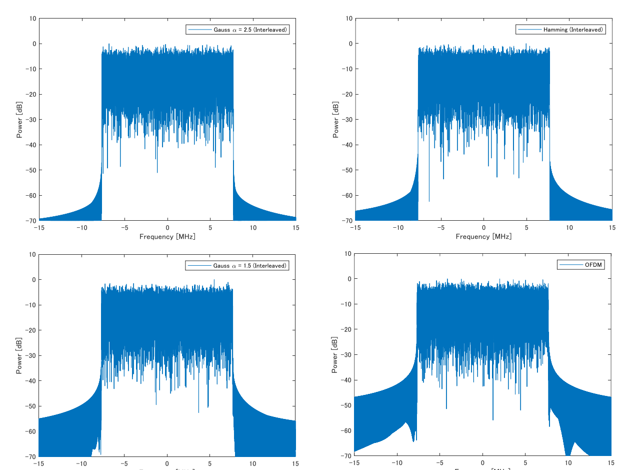

Figure 10 Spectrum of proposed pre-coded windowed OFDM and conventional OFDM