Time-Compressed Synchronization Sequence for Future Spectrally Efficient Transmission Schemes

Myungsup Kim, Jiwon Jung, and Ki-Man Kim
Dept. of Radio Communication Engineering
Korea Maritime and Ocean University
Busan, Korea
myungsup@kaist.ac.kr, {jwjung, kimkim}@kmou.ac.kr

Abstract— In this paper, we propose a technique for generating a synchronization sequence for estimating time and frequency offsets for spectrum efficient transmission schemes. The synchronization sequence should have excellent auto-correlation characteristics without deteriorating the efficiency of various spectrum efficient transmission schemes. Since the columns of the discrete filter matrix obtained from the coefficients of the binomial function have a very short finite length and limit the band, the performance can be accurately estimated. In this paper, we propose a scheme for generating a sequence with excellent auto-correlation characteristics while being band-limited and, for example, verify the accuracy of the scheme.

***Keywords*—BFDM, binomial, emission, FBMC, frequency, GFDM, out of band**

I. INTRODUCTION

The transmission scheme for future communication is highly likely to develop into a scheme with spectral efficiency. This research direction is to improve the current OFDM envelope because it is a rectangular shape [1]. For the past 10 years, studies for out-of-band power cutoff (OoBE) have been actively conducted [2-5]. Efforts to reduce OoBE are expected to continue in the future.

One of the important technologies among transmission technologies is synchronization technology. Visual information can be extracted so that transmission symbols can be safely received, and a frequency offset can be estimated based on this. The existing LTE scheme uses the Zadoff-Chu (ZC) sequence [6]. The reason is because of its excellent autocorrelation properties. However, it is not suitable for continued use of such synchronization sequences in the future. The reason is that the ZC sequence has the property of spreading the spectrum.

In this study, we propose an efficient filtering scheme for ZC sequences. In the frequency domain, each column of the proposed filter matrix is obtained from the coefficients of the binomial polynomial. After filtering by multiplying the input data symbol vector with the filter matrix, a synchronization sequence is obtained in the time domain through IFFT. The synchronization sequence obtained in this way has an autocorrelation property superior to that of the original ZC. In addition, it shows that it has good crosscorrelation without increasing the spectrum by sending the overlapped synchronization sequence for frequency offset estimation.

Section II describes the method of generating a synchronization sequence, frequency response characteristics, sequence envelope, and autocorrelation characteristics. Section III describes the concept and description of a time-compressed sequence, and analyzes the crosscorrelation characteristics of sequence generation, for example. By overlapping the two sequences by 50%, the transmission time of the synchronization sequence can be shortened. However, it is shown that the crosscorrelation characteristics of the two sequences do not interfere with each other. Section IV describes the statistical properties of the proposed synchronization sequence. Cramer-Rao Bound (CRB) for the received synchronization sequence is obtained. Also, when the synchronization sequence is detected by the correlator, it is confirmed that the correlator is the minimum variance estimator because the variance has the minimum value of the CRB. Section IV summarizes the results of this study and mentions its potential for future communications.

II. SYNCHRONIZATION SEQUENCE

A. Sequence Generation

The binomial frequency division multiplexing (BFDM) [5] synchronization signal is a narrowband signal in which out-of-band power leakage is suppressed in neighboring channels. Therefore, the synchronization signal for detecting a symbol and detecting the frequency offset must also be composed of a BFDM signal. The generated sequence is as follows:

$$\mathbf{s} = \mathbf{F}^{H}\mathbf{A}\mathbf{d}, \tag{1}$$

where $\mathbf{F}$ is a DFT matrix, $\mathbf{A}$ is a filtering matrix, and $\mathbf{d}$ is a predetermined random data symbol vector. Thus, the vector $\mathbf{s}$ entries become an arbitrary sequence with a bandwidth determined by $\mathbf{A}$. Generating $\mathbf{s}$ with an equation such as (1) is to maintain consistency with the scheme of transmitting data in the frequency domain and data in the existing frequency domain. Sequence transmission consists of N samples in order from $\mathbf{s}[1]$ to $\mathbf{s}[N]$. Fig. 1 shows the role of a data vector and two matrices to generate a synchronization sequence.

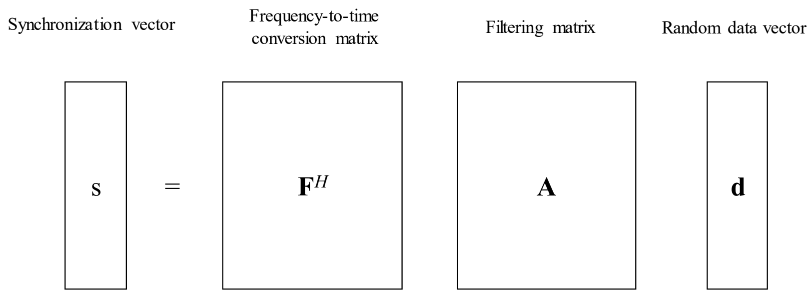


Fig. 1. Synchronization vector generation structure

The entries of vector $\mathbf{d}(\in\mathbb{C}$ are composed of the even ZC sequence [6]

$$d[n] = \exp\left(-j\frac{\pi R(n-1)^2}{M}\right), \quad 1 \le n \le M. \tag{2}$$

B. Practical Application

Fig. 2 shows the autocorrelation characteristics of the ZC sequence. Compared to the center, the residual correlation characteristic also shows a small level of noise level.

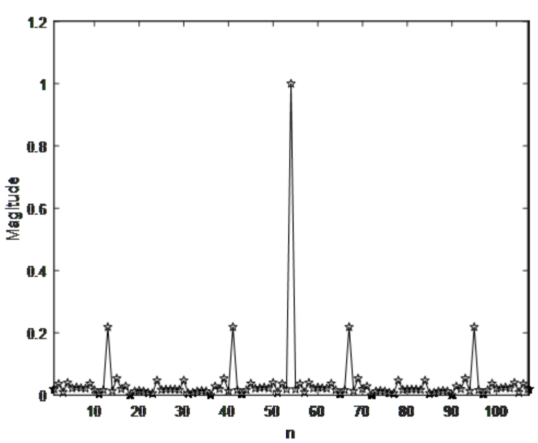


Fig. 2. Autocorrelation characteristics of vector **d**. (K = 2, R = 25, M = 52, N = 64)

The matrix **A** has a form in which the same column vector is shifted down by one entry.

$$\mathbf{A} = \begin{bmatrix} a_0 & & & \\ \vdots & & & \\ a_K & \vdots & & \\ & a_K & \ddots & \\ & & & a_0 \\ & & & \vdots \\ & & & a_K \end{bmatrix} \tag{3}$$

$$\mathbf{F} = \left(\frac{\omega^{mn}}{\sqrt{N}}\right)_{m,n=0,\cdots} \tag{4}$$

where $\omega = e^{-2\pi j/N}$, $j^2 = -1$, and the entry of $\mathbf{a}_K$ is

$$a_i = C_K(-1)^i\binom{K}{i}, \quad i = 0, 1, 2, \cdots \tag{5}$$

where C_K is a normalized factor that makes it the sum of the squares of each column of $\mathbf{A}$ and is expressed as

$$C_K = \left(\sum_{i=0}^{K}\left((-1)^i\binom{K}{i}\right)^2\right)^{-\frac{1}{2}} = \binom{2K}{K}^{-\frac{1}{2}}. \tag{6}$$

The number of coefficients depends on the order K and the number is K+1. Each column of matrix **A** serves as a filtering with K+1 taps. Since $\mathbf{A}$ and $\mathbf{F}$ are deterministic matrices, the randomness of the vector s depends on the vector $\mathbf{d}$, but the envelope of the synchronization sequence depends on the contents of $\mathbf{A}$ and $\mathbf{F}$.

When the m-th column of matrix A is expressed in the time domain, it is as follows:

$$s_m(n) = C_K e^{j\frac{2\pi(m-1)}{N}} \sum_{i=0}^{K} a_i e^{j\frac{2\pi i n}{N}}, \quad n = 0, 1, 2, \cdots \tag{7}$$

Since the column of $\mathbf{A}$ is obtained from the binomial polynomial, the transmission sequence by the m-th column of matrix $\mathbf{A}$ can be expressed as

$$\begin{aligned} s_m(n) &= C_K e^{j\frac{2\pi(m-1)n}{N}} \left(1 - e^{j\frac{2\pi n}{N}}\right)^K \\ &= (-j2)^K C_K e^{j(2m-2+K)\frac{\pi n}{N}} \sin^K \frac{\pi n}{N}. \end{aligned} \tag{8}$$

The envelope function of $s_m(n)$ is

$$|s_m(n)| = 2^K \binom{2K}{K}^{-\frac{1}{2}} \sin^K \frac{\pi n}{N}, \quad n = 0, 1, \cdots \tag{9}$$

From (9), it can be seen that the envelope function is constant regardless of the position of the column of matrix $\mathbf{A}$. That is, they have the same envelope for all rows, and the size is only dependent on the order K. Fig. 3 shows the envelope according to the order K given by (9), and it can be seen that the higher the order, the more the power is concentrated in the center.

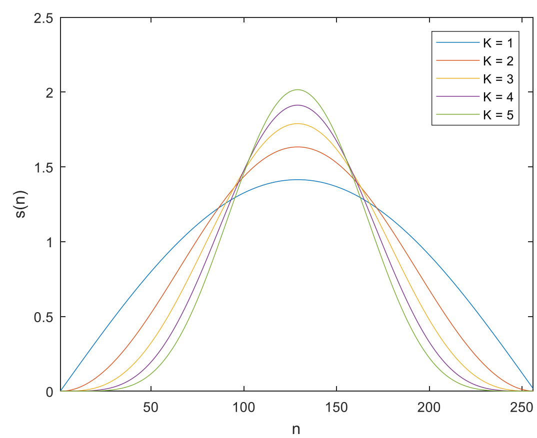


Fig. 3. Envelop functions according to order K

To see the frequency response characteristics, we change the matrix $\mathbf{A}$ as $\mathbf{B}$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0}_\alpha & \\ \mathbf{A}^c & \\ \mathbf{0}_\beta & \\ & \mathbf{A}^c \end{bmatrix}, \tag{10}$$

where $\mathbf{0}_\alpha \in \mathbf{R}^{1\times M}$, $\mathbf{0}_\beta \in \mathbf{R}^{(N-M-1)\times M}$ and $c = \dfrac{N-M}{2}$.

Fig. 4 shows the frequency response characteristics of matrix $\mathbf{B}$. It has been shown that the magnitude of the signal at the spectral boundaries is significantly decreasing.

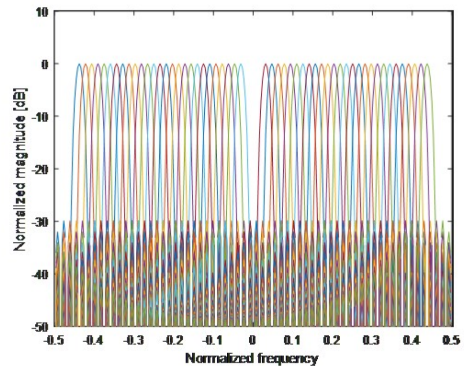
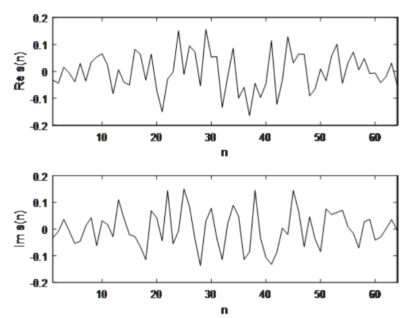
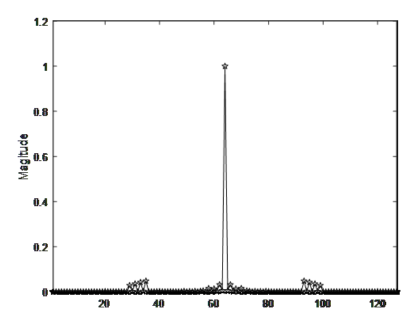


Fig. 4. Frequency response characteristics ($K = 2$, $R = 25$, $\mathbf{B} \in \mathbf{R}^{64\times 52}$.)

Fig. 5 shows the characteristics of the generated sequences. Fig. 5 (a) shows the characteristics of the synchronization sequence in the time domain. Since it has a small value at both ends of the sequence and the power of the signal is large at the center, it can be seen that a lot of power is transmitted at the center of the sequence. Fig. 5 (b) shows the autocorrelation characteristics of the filtered synchronization sequence. Compared to Figure 2, which is a characteristic without filtering, the height of the peaks except for the highest peak in the center is much smaller.



(a)



(b)

Fig. 5. Filtered synchronization sequence (K = 2, R = 25, $\mathbf{B} \in \mathbf{R}^{64\times52}$.): (a) Filtered synchronization sequence in the time domain, (b) Autocorrelation property.

III. TIME-COMPRESSED SEQUENCE

In general, in the synchronization sequence, when the same pattern is continuously transmitted and received through a channel, a frequency offset generated between a transmitter and a receiver can be estimated. In this case, the sequence to be used must have sufficiently random characteristics and thus have excellent correlation characteristics. In Fig. 6(a), the same sequence is repeatedly transmitted, and the power is concentrated in the center and the transition of the envelope is smooth. Figure 6(b) shows that the two sequences are overlapped. In addition, the core of an important technology is to shorten the transmission time because the size of both ends of the two sequences is small, so even if the two sequences are overlapped, the size of the entire sequence does not change. At this time, it is important to note that the sequence should be designed so that the overlapped portion minimizes mutual interference between the two sequences.

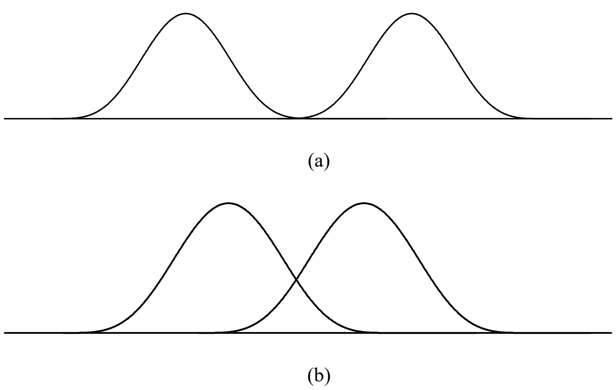


Fig. 6. Concept of time-compressed sequence: (a) Separated type, (b) Overlapped type.

The sequence represented in Fig. 6(b) is

$$
\begin{aligned}
ss[n] &= s[n] + s\left[n - \frac{N}{2}\right], \quad n = 1, 2, \cdots \\
s[n] &= 0, \quad n = -\frac{N}{2} + 1, \cdots \\
s[n] &= 0, \quad n = \frac{N}{2} + 1, \cdots
\end{aligned}
\tag{11}
$$

It is shown that the sequence in (11) can be expressed as the sum of the original sequence and the $N/2$ delayed sequence. In other words, this is an example of overlapping 50% of the sequence length. In order to send two sequences, $2N$ samples must be sent, but in this example, if only $3N/2$ samples are sent, two sequences are sent, which is the result of compressing the time by $N/2$. In the same vein, as the order K is higher, the sequence power can be concentrated to the center, so the compression efficiency can be increased by placing more elements of overlap.

Fig. 7 shows the transmission of two consecutive sequences. It is shown that the envelopes of both real and imaginary signals have the same envelope shape. It is shown that the two sequences are completely separated in time and do not interfere with each other in time. In order to estimate the frequency, offset, such a scheme must have a synchronization pattern having a short length, but since such a sequence causes spectrum spreading, it is not suitable as a synchronization acquisition scheme among future communication schemes.

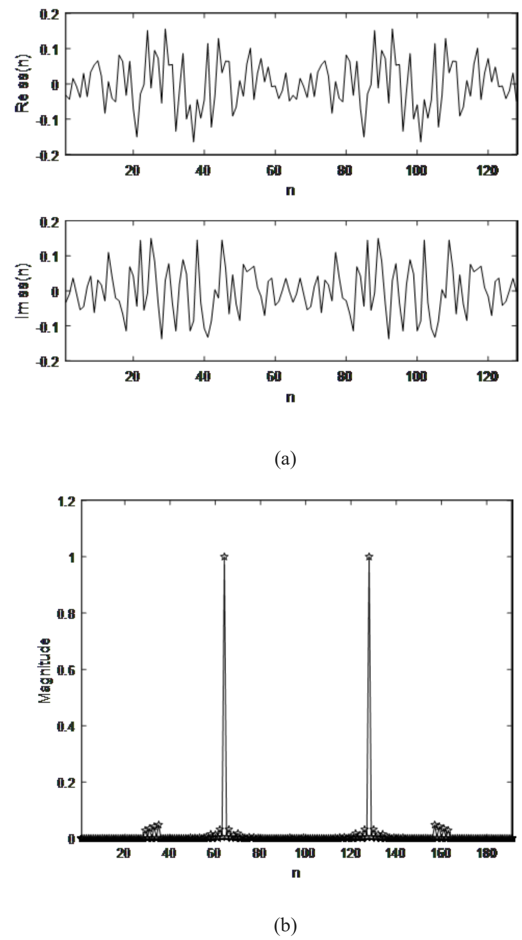
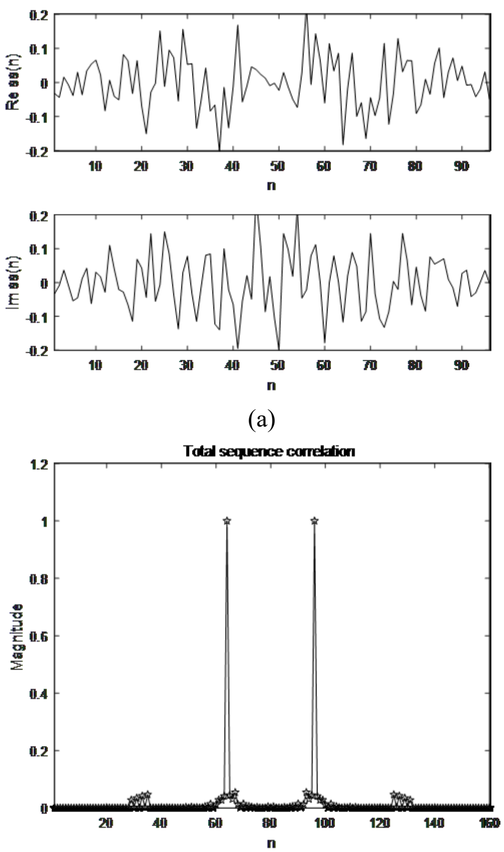


Fig. 7. Characteristics for a composite synchronization sequences (K = 2, R = 25, $\mathbf{B} \in \mathbf{R}^{64\times52}$): (a) Representation of the sequence in the time domain, (b) Crosscorrelation.

Fig. 8 shows the time-compressed scheme. The two sequences are temporally separated. This scheme takes the scheme of estimating the channel by obtaining two independent pulses. However, there is a disadvantage in that the range in which the frequency offset can be estimated is narrow.



(a)

(b)

Fig. 8. Two consecutive sequences (K = 2, R = 25, $\mathbf{B} \in \mathbf{R}^{64\times52}$): (a) Superimposed synchronization sequences in the time domain, (b) Crosscorrelation characteristics of superimposed synchronization sequences.

IV. STATISTICAL PROPERTIES

A. CRB

The vector $\mathbf{s}$ of (1) is received over the AWGN channel as

$$\mathbf{r} = \mathbf{s} + \mathbf{n}. \tag{12}$$

where $\mathbf{n}$ is the noise vector.

The vector $\mathbf{r}$ can be expressed on a sample basis as

$$r[k] = s[k] + n[k], \quad k = 1, 2, ..., N. \tag{13}$$

The pdf of $\mathbf{r}$ is

$$f_{\mathbf{r}}(\mathbf{r}) = \frac{1}{\left(\sigma\sqrt{2\pi}\right)^N} \exp\left(-\frac{1}{2\sigma^2}\sum_{k=1}^{N}\left(r[k] - s[k]\right)^2\right). \tag{14}$$

Taking log in (13), the log likelihood function becomes

$$l(s[k]) = -\frac{1}{2\sigma^2}\sum_{k=1}^{N}\left(r[k] - s[k]\right)^2. \tag{15}$$

Differentiating (14) twice, we have

$$\frac{\partial^2}{\partial\left(s[k]\right)^2} l\left(s[k]\right) = -\frac{N}{\sigma^2}. \tag{16}$$

Therefore, the variance of $\mathbf{r}$ is

$$\mathrm{var}(\mathbf{r}) \geq \frac{\sigma^2}{N}. \tag{17}$$

B. Detection by Correlator

The received sample $r[k]$ is multiplied by a known sample $v^*[k]$ as

$$\begin{aligned} z[k] &= v^*[k]r[k] \\ &= v^*[k]\left(v[k] + n[k]\right) \\ &= v^*[k]v[k] + v^*[k]n[k] \\ &= \gamma[k] + q[k], \end{aligned} \tag{27}$$

where $\gamma[k] = v^*[k]v[k]$ is a deterministic sample, since $v[k]$ is a predetermined synchronization sequence..
We have

$$\sigma_q^2 = \sigma_n^2. \tag{28}$$

The pdf of the output of a correlator can be written as

$$f_z(\mathbf{z}) = \frac{1}{\left(\sigma\sqrt{2\pi}\right)^N} \exp\left(-\frac{1}{2\sigma_n^2}\sum_{k=1}^{N}\left(z[k] - \gamma[k]\right)^2\right). \tag{29}$$

As in subsection A above, we have the CRB as

$$\mathrm{var}(\mathbf{z}) \geq \frac{\sigma_n^2}{N}, \tag{30}$$

which coincides with (17), so the correlator is the minimum variance estimator. Therefore, the correlator used in the discussion in this paper proves to be an optimal detector.

V. 5 | CONCLUSION

Future communication methods are evolving to maximize transmission efficiency in the time domain and frequency domain. Accordingly, in order to support this, synchronization schemes that do not degrade the performance of the new transmission scheme in the time and frequency domains are required. Since the column vector of the filtering matrix is obtained from the coefficients of the binomial polynomial, it is easy to express in the frequency domain and the performance analysis can be performed relatively easily. Depending on the application, spectral formation and autocorrelation can be controlled by adjusting the order of the binomial polynomial. As a result of analyzing the probabilistic characteristics, it was confirmed that the detection method by correlator is the least variance estimation method. The proposed scheme maintains a good correlation in a band-limited environment, and is expected to be applied to various transmission schemes emerging for communication development in the future.

VI. CONCLUSION

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