BER Minimization by User Pairing in Downlink NOMA Using Laser Chaos-Based MAB Algorithm

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***Abstract*—Non-Orthogonal Multiple Access (NOMA) is the technology that allows multiple users' downlink communications to be transmitted in the same resource block by proper user pairing. In realizing real-time operations, an ultrafast pairing decision scheme is required. Previous studies have shown that decision making using laser chaos is ultra-fast and effective as a Multi-Armed Bandit (MAB) algorithm. In this paper, we demonstrate user pairing using laser chaos-based MAB algorithm on the basis of the bit error rate of the physical layer. That is, we define the conditions for successful or failed communication by bit error, leading to benefits from the efficient decision-making ability of the laser chaos-based MAB strategy. The numerical results show that the proposed method provides better performances than conventional ones, C-NOMA and UCGD-NOMA.**

***Index Terms*—Non-Orthogonal Multiple Access (NOMA), Laser Chaos, Multi-Armed-Bandit Problem, MAB algorithm, Bit error, User Pairing**

I. Introduction

With the advent of 5G, the upcoming years will see an explosive growth of mobile data traffic and a dramatic increase in the number of mobile devices, calling for the introduction of revolutionary wireless technologies to sustain the ever-increasing demand for bandwidth and services [1].

Non-orthogonal multiple access (NOMA) has been recognized as an essential technology for improving connectivity, spectral efficiency, cell-edge throughput, and user fairness in the fifth generation and beyond wireless networks. NOMA allows multiple users to use the same frequency band and time by assigning different power [2]. On the other hand, at the receiver side, multiuser-detection (MUD) algorithms such as successive-interference-cancellation (SIC) are implemented to identify specific signals [2]. User pairing is a way to select two users who are multiplexed in the same resource block. There is a growing interest in this user pairing [3] [4]. Pairing is important in NOMA for a variety of reasons. Reasons include the fact that the larger the channel gain difference between users, the smaller the impact of SIC execution errors on the system performance, and the different throughput that can be obtained from pairing [3] [4] [5]. It has been found that the pairing schemes proposed in [4] do not provide optimal pairing [5].

Reinforcement learning methods, e.g., deep reinforcement learning methods and (Mulit-Armed Bandit) MAB methods, are often used to solve problems in wireless communication [6] [7]. In [6],[7], Q-Learning and deep Q-Learning are applied to NOMA respectively. However, in order to apply these reinforcement learning methods, state information such as the user's location is required. Therefore, it takes time to obtain the state information, which results in delays. The MAB algorithm, on the other hand, can make decisions without state information, so it can be applied to real-time operation. User pairing problems and some other problems in wireless communication are sometimes treated as MAB problems [5] [8] [9]. The MAB problem is the problem of finding the machine with the highest reward probability from multiple slot machines with unknown reward probabilities [9].

In [10], Naruse et al. solves the MAB problem by using chaotic oscillatory waves generated by a semiconductor laser. Decision making is very fast using the MAB algorithm based on laser chaos [10]. In MAB problems, it is important to explore alternatives in order to find the best choice [11]. In [12], it is possible to generate a good quality random bit sequence at a very high bit rate. It has been shown that the laser chaotic time series can be used to solve the MAB problem very fast [10], [13]. It has been shown that scalable decision making up to 64-arm bandit problems is possible [13]. The MAB algorithm is based on laser chaos, and it selects one slot machine by successive large and small comparisons between the set threshold and the sampled data of the laser chaos waveform. Then, depending on the results of playing

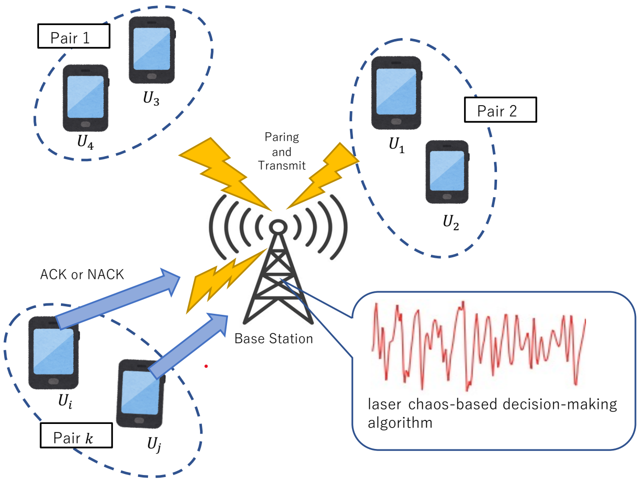

Fig. 1. System model.

the selected slot machine, the threshold is adjusted to make a better choice. If the option to be finally selected is mapped to a bit string, one slot machine is finally selected by setting the result of the comparison between the threshold and the sampled data of the laser chaotic waveform as "0" or "1".

Duan et al. propose a pairing method using MAB algorithm based on laser chaos for NOMA systems [5]. The slot machines are associated with user pairing options in MAB problem, and the selected slot machine by the laser chaos-based decision maker means the selected pairing. In [5], the rewards of the MAB problem were defined by communication throughputs. This way, however, requires a not-so-small time duration to realize the rewards; therefore, the fast decision ability of the laser chaos-based MAB algorithm is not unfortunately fully utilized. Furthermore, the system model in [5] is too simple to evaluate the exact performance in NOMA systems.

In this paper, to overcome such limitations, the pairing problem is considered by examining bit error rate (BER) in adapting the MAB algorithm based on laser chaos. We compare the BER performance of the proposed method with conventional paring strategies, such as C-NOMA [4] and UCDG-NMA [4].

The rest of the paper is organized as follows. In Section II, we provide the system model and the problem formulation. In Section III, we introduced the operating principle of the laser chaos-based MAB algorithm. In Section IV, the pairing method in the NOMA system based on laser chaos-based MAB algorithm is proposed on the basis of channel quality or BER. Section V shows numerical demonstrations of the proposed method. Section VI concludes the paper.

II. SYSTEM MODEL

In this paper, we consider a downlink single cell system where one base station (BS) provides services to multiple users. Fig. 1 shows an overview of the system model of the NOMA system under study. All transmitters and receivers have one antenna each. Let $U{=}\{U_1, U_2, \cdots, U_n, \cdots, U_N\}$ be defined as the set of N users in the circular cell and $K{=}\{1, 2, \cdots, k, \cdots, N/2\}$ be defined as the index of each pair. N is the total number of users, which is an even number. We assume that the distance ordered as $d_1{\le}d_2{\le}\cdots{\le}d_N$ where d_n is the distance from the base station (BS) to the n^{th} user. The multiplexed signal x_k to the k^{th} pair is defined as follows [14]:

$$x_k = \sqrt{a_k P_k} x_k^i + \sqrt{(1-a_k)P_k} x_k^j \tag{1}$$

where x_k^i and x_k^j are the signals for the i^{th} and j^{th} users that from the pair k ($d_i{\le}d_j$). a_k is the power allocation factor for the k^{th} pair. P_k is the power value assigned to each pair for the k^{th} pairs.

At the receiver of the k^{th} pair, the received signal y_k is defined as follows [14]:

$$y_k = d_n^{-\lambda} \hbar_n x_k + w_n \tag{2}$$

where $d_n^{-\lambda}$ is the pathloss between BS and n^{th} user and λ is the pathloss exponent, is the Rayleigh fading of n^{th} user and w_n is the additive white gaussian noise (AWGN).

The base station performs user pairing using the MAB algorithm based on laser chaos and sends data to the user according to the pairing. After receiving and decoding the data, the user detects the bit errors and returns a response (ACK or NACK) to the base station depending on the number of bit errors. Let $bit\ error_n$ be defined as the number of bit errors detected by the n^{th} user and $r_n = \{0, 1\}$ be defined as the n^{th} user's response to the base station. $r_n = 1$ means that the n^{th} user sent ACK to the base station. $r_n = 0$ means that the n^{th} user sent NACK to the base station. At the user side, the specific signal of each pair is restored via the SIC.

Our goal is to maximize the communication success rate by optimizing user pairing, which can be expressed as follows.

$$\max_b \sum_{n=1}^{N} r_n. \tag{3}$$

where b is the pairing option, which refer to which user to pair with. The larger of the value $\sum_{n=1}^{N} r_n$ in Eq. (3) is, the higher communication success rate that the NOMA system can obtain while the lower of the users' BER will be. The maximum value of $\sum_{n=1}^{N} r_n$ in Eq. (3) is N. At that time, all users communicate successfully while the BER is minimized. The obtained pairing option is the optimal user pairing solution of our formulated problem when $\sum_{n=1}^{N} r_n = N$. In order to achieve this goal, optimal user pairing must be performed. In this paper, the optimal user pairing is conducted using the MAB algorithm based on laser chaos.

III. DECISION MAKING AS A MAB ALGORITHM BASED ON LASER CHAOS

Laser chaos is the chaotic output produced by a semiconductor laser. The following methods are used to generate laser chaos. The oscillation of lasers becomes unstable and leads to chaos when a portion of the output light is fed back to the

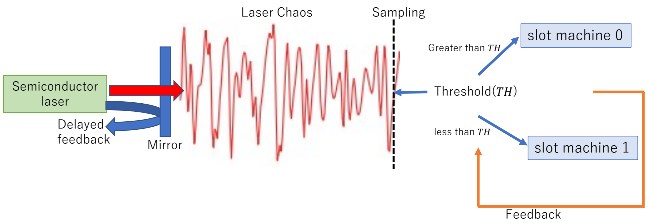

Fig. 2. MAB algorithm based on laser chaos.

laser cavity after a certain delay via an externally arranged mirror [9].

Fig. 2 schematically outlines the decision making based on MAB algorithm using laser chaos time-series generated by a semiconductor laser [9]. The principle of decision making based the laser chaos can be summarized as follows.

First, the initial value of the threshold is set. Next, a decision is made by comparing the sampled laser time series data with the threshold value. If the sampled time series data is greater than the threshold value, slot machine 0 is selected. Otherwise, slot machine 1 is selected. The threshold is adjusted according to whether the selected slot machine can be executed and rewarded, so that slot machines with higher reward probability will be selected in order to increase the reward obtained in the future.

The threshold $TH(t)$, which is compared with the sampling of laser chaos in step t, is given by

$$TH(t) = k \times \lfloor TA(t) \rfloor \tag{4}$$

where $TH(t)$ is the threshold adjuster value at step t, $\lfloor TA(t) \rfloor$ is the closest integer to $TA(t)$ rounded to zero, and k is a constant to control the range of $TH(t)$. $\lfloor TA(t) \rfloor$ is $-Z, \cdots, -1, 0, 1, \cdots, Z$, where Z is a natural number. Thus, the number of thresholds is $2Z+1$. $\lfloor TA(t) \rfloor$ is limited to the range of $-kZ$ to kZ.

The threshold adjuster TA is updated according to the following:

$$TA(t+1) = \begin{cases} \pm\Delta + \alpha TA(t), & \text{if rewarded. (a)} \\ \mp\Omega + \alpha TA(t), & \text{otherwise. (b)} \end{cases} \tag{5}$$

where α $(0 \leq \alpha \leq 1)$ is a forgetting rate to control the impacts of past experiences, Δ is a reward and Ω is a penalty. In the case of a reward, i.e., if a benefit is obtained by playing the selected slot machine, the threshold adjustment value $TA(t)$ is updated according to Eq. (5a). If it is not a reward, i.e., you did not get a benefit by playing the selected slot machine, the threshold adjustment value $TA(t)$ is updated according to Eq. (5b).

Ω is a value based on past choices and benefits. Let, S_i be the number of times slot machine i was selected in step t. Let L_i be the number of times you played the selected slot machine i in step t and obtained a benefit. At this time, the estimated reward probability of i^{th} slot machine P_i is given by:

$$P_i = \frac{L_i}{S_i} \tag{6}$$

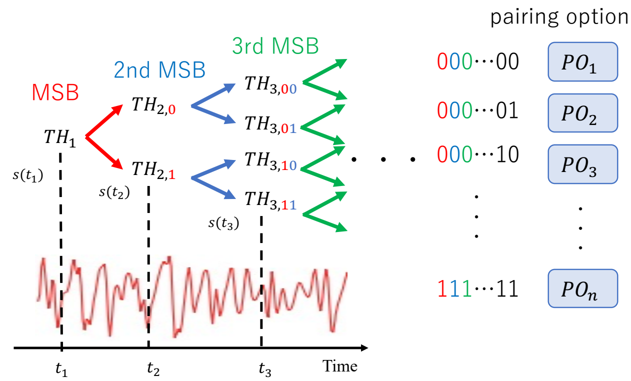

Fig. 3. User pairing using MAB algorithm based on laser chaos.

In the two-armed MAB problem, we use the estimated reward probability in Eq. (6) to define Ω as follows:

$$\Omega = \frac{P_0 + P_1}{2 - (P_0 + P_1)} \tag{7}$$

If it is not a reward, i.e., you did not get the benefit by playing the selected slot machine, the threshold adjustment value $TA(t)$ is updated according to Eq. (5b), using Eq. (7).

IV. USER PAIRING BY LASER CHAOS-BASED MAB ALGORITHM BASED ON BIT ERROR

In this section, we describe MAB algorithm based on a laser chaos for user pairing in NOMA.The proposed method is a reinforcement learning algorithm that reaches the optimal pairing by repeating the following process: determine the pairing options by the MAB algorithm based on laser chaos, communicates with the pairings, and adjusts the threshold based on the communication results. Fig. 3 shows the structure of MAB algorithm based on laser chaos for the NOMA user pairing problem, where the slot machines in the MAB problem are treated as user pairing options. The user pairing options are selected by comparing the sampled data from the laser chaos time series with a threshold value. The threshold is adjusted based on the reward of communicating with the selected user pairing option. In Fig. 3, PO_n is the n^{th} option selected. The next step is to describe the rewards and penalties needed to adjust the threshold.

Next, we explain that the detailed time series and threshold comparison method. In the laser chaos-based MAB algorithm, the identity of the pairing option to be selected can be determined bit by bit in a pipelined fashion from the most significant bit (MSB) to the least significant bit (LSB). For each bit, the decision is based on a comparison between the measured chaotic signal level and a specified threshold value. First, we determine the most significant bit (MSB). At $t = t_1$, compare the level of the chaotic time series with the threshold value TH_1. If the time series is greater than or equal to the threshold value, the chosen option is 0, which we denote as D_1 (MSB) $= 0$. Otherwise, the result will be 1 $(D_1 = 1)$.

Then, we decide the second most significant bit. In the case the MSB is determined by $D_1 = 0$ and compare the level of the time series with the threshold value $TH_{2,0}$. The first number 2 in the $TH_{2,0}$'s subscript indicates that the threshold value is related to the second most significant bit, and the second number 0 in the subscript indicates that the previous decision was 0 ($D_1 = 1$). If the time series is greater than or equal to the threshold $TH_{2,0}$, then the second most significant bit is 0 ($D_2 = 0$), otherwise it is 1 ($D_2 = 1$).

Finally, we have to determine the least significant bit. According to the above rules, threshold value comparison finishes when all L bit information of the specified option is determined. If $L = 4$, the result of the 4^{th} comparison is the least significant bit of the combination to be selected. The update formula of the threshold adjuster value (TA) is expressed as follows:

$$TA_{L,M_1,M_2,\cdots,M_{(L-1)}}(t+1) = \begin{cases} \pm\Delta + \alpha TA_{(L,M_1,M_2,\cdots,M_{(L-1)})}(t), & \text{if rewarded. (a)} \\ \mp\Omega + \alpha TA_{(L,M_1,M_2,\cdots,M_{(L-1)})}(t), & \text{otherwise. (b)} \end{cases} \quad (8)$$

In Eq. (8a), the reward is given when $M_L = 0$ or 1, $D_1 = M_1, \cdots, D_{L-1} = M_{L-1}$ are determined. In Eq. (8b), the reward is not given when $M_L = 0$ or 1, $D_1 = M_1, \cdots, D_{L-1} = M_{L-1}$ are determined. In this time, $D_1 D_2 \ldots D_L$ is the selected option associated with the bit sequence. α ($0 \leq \alpha \leq 1$) is a forgetting rate to control the impacts of past experiences.

It is mentioned that each user responds to the base station according to the success or failure of the communication in Section II and the conditions of success or failure are explained below. After receiving and restoring the data, each user detects the bit error. The n^{th} user compares this $bit\ error_n$ with the n^{th} user's bit error reference value err_n to determine the success or failure of communication. If $bit\ error_n \leq err_n$, the communication is regarded as successful; ACK is sent to the base station. In the case of $bit\ error_n > err_n$, the communication is regarded as failed; NACK is sent to the base station. The equation is as follows:

$$r_n = 1 \quad \text{if } bit\ error_n \leq err_n \quad (9)$$

$$r_n = 0 \quad \text{if } bit\ error_n > err_n \quad (10)$$

The base station rewards and penalties the threshold according to the number of ACKs received. If the number of ACKs received at the base station is greater than or equal to X, give a reward, otherwise give a penalty.

V. SIMULATION RESULTS

In this section, we present the numerical results to evaluate the performance based on BER. Besides, we compared our proposed method Laser Chaos Decision Making (LCDM)-NOMA to C-NOMA [4] and UCGD-NOMA [4]. In C-NOMA and UCGD-NOMA, users are first divided into two categories according to their distance from the base station: "near area" and "far area". C-NOMA's pairing scheme is that pairs the closest user to a base station in the short-range region with the farthest user to a base station in the long-range region, the second closest user in the short-range region with the second farthest user in the long-range region, and so on, and pairs other users as well. UCGD-NOMA's pairing scheme is that pairs the user who is closest to a base station in the short-range area with the user who is closest to a base station in the long-range area, the second closest user in the short-range area with the second closest user in the long-range area, and so on, and pairs other users as well. In our simulation, we consider 10 users ($N = 10$) and a cell with a radius of 1000 m where users are arranged randomly. Path loss exponent is set as 2.7. The power allocation is fixed, what it means that all pairs are allocated 30 dBm regardless of pairing option and the power allocation factor is $a = 0.1$. We assume that the standard is $k = 128$ and $Z = 1$ in MAB algorithm based on laser chaos. In addition, the forgetting rate α is 1.0 and Δ is 1.0 in that one. data bit per user is 256 bit. $x_k^i$and x_k^j is a signal of "data bit per user" modulated by QPSK and made into OFDM symbols by IFFT. The number of bit errors to be tolerated, err_n, can be fixed or adaptive. In the fixed case,

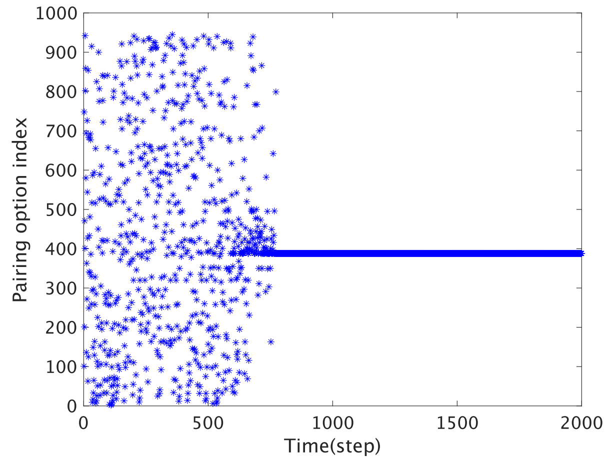

Fig. 4. Pairing option selected by MAB algorithm based on laser chaos at each step.

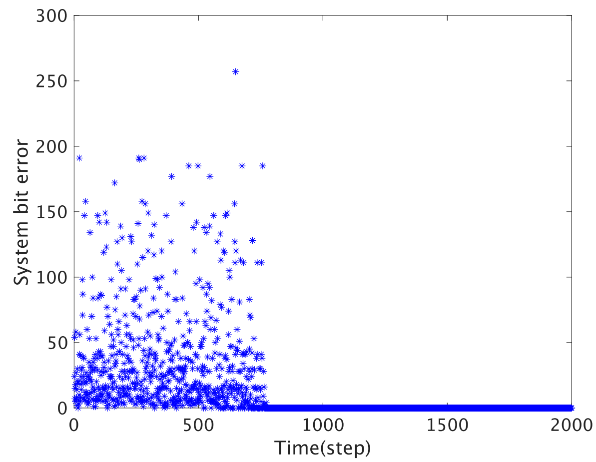

Fig. 5. *System bit error* for the pairing option selected at each step.

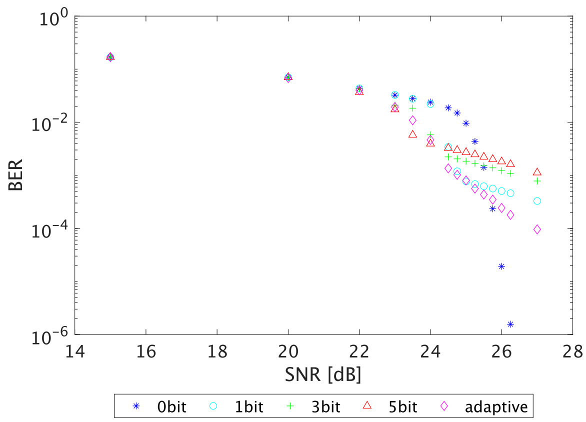

Fig. 6. BER under different err settings when the cell radius and the path loss exponent are 1000 m and 2.7, respectively.

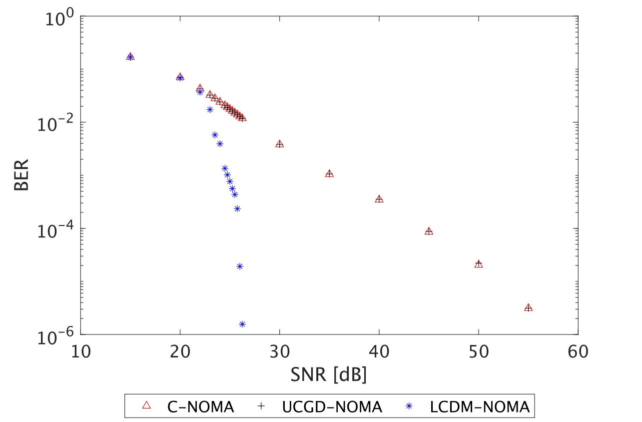

Fig. 8. BER comparison with LCDM-NOMA and C-NOMA and UCGD-NOMA.

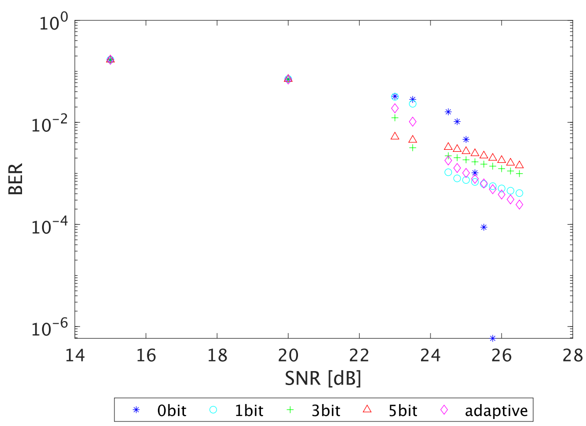

Fig. 7. BER under different err settings when the cell radius and the path loss exponent are 500 m and 3.5, respectively.

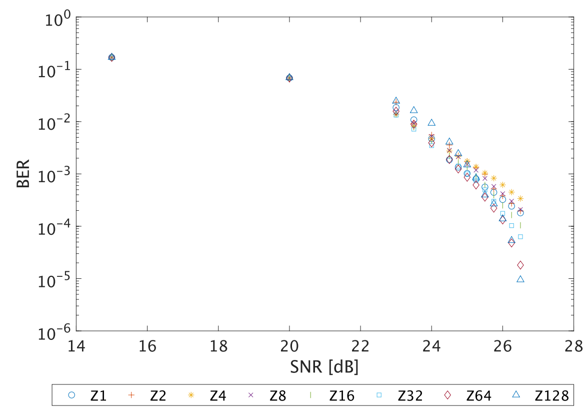

Fig. 9. BER with different number of thresholds.

err_n is the same for all steps. In the case of adaptive, err_n is the average of the last five bit errors of its own.

In this simulation, 2000 steps are used as the threshold learning step, and this 2000^{th} step is executed 10000 times. 2000^{th} step pairing option is considered to be a better one on the basis of the MAB algorithm in Section IV. We show the effectiveness of the proposed method by evaluating the BER of this 2000^{th} step. The BER is calculated as follows:

$$BER = \frac{\sum_{r=1}^{10000} \sum_{n=1}^{N} bit\ error_{n,2000^{\text{th}}}}{data\ bit\ per\ user \times N \times 10000} \tag{13}$$

where $biterror_{n,2000^{\text{th}}}(r)$ is the bit error at the 2000^{th} step of the n^{th} user in the r^{th} execution.

Fig. 4 shows that the results of the pairing option selected by MAB algorithm based MAB laser chaos in one run. The horizontal axis of Fig. 4 is the number of steps, and the vertical axis is the pairing option index. Fig. 4 illustrates the pairing selected according to MAB algorithm based on laser chaos is changing before about 750 steps. In this run, after about 750 steps, the proposed scheme can converge to the 388^{th} pairing option and no longer change.

Fig. 5 shows that the variation in the number of $system\ bit\ error$ at each step in one run. $system\ bit\ error$ is the sum of $bit\ error_n$ in each step, expressed in a formula, $system\ bit\ error = \sum_{n=1}^{10} bit\ error_n$. From Fig. 4, we can see that the pairing options converge to one, so the system bit error also converges and its value is zero.

Fig. 6 shows that the BER after 2000 steps to each SNR. In Figure 6, "0 bit, 1 bit, 3 bit, 5 bit" is the result when err_n is fixed at 0, 1, 3, or 5 bits for all users in all steps and all runs, respectively. "adaptive" is the result when err_n is set to the average of the last five bit errors of the user. It can be seen that SNR is more higher, BER becomes more smaller. Furthermore, it can be seen that the optimal err_n differs depending on the SNR. That is why we have to consider setting err_n.

Fig. 7 shows the BER when the cell radius and path loss exponent are set to 500 m and 3.5, respectively. Other parameter settings are the same as in Fig. 6. From Fig. 7, we can get the same conclusion as that from Fig. 6, which shows that the variation trend of BER under different err_n settings do not change with the cell radius and path loss exponent.

Fig. 8 shows that a BER comparison to C-NOMA and UCGD-NOMA. We observe from Fig. 8 that the proposed LCDM-NOMA achieves a smaller BER than the C-NOMA and UCGD-NOMA. Therefore, we conclude LCDM-NOMA is better than C-NOMA and UCGD-NOMA when BER is concerned.

Fig. 9 summarizes the effect of the number of thresholds on the BER in LCDM-NOMA and the resultant bit error rate. The number of thresholds is given by $2Z + 1$ where Z is a natural number. The fewer the number of thresholds, the quicker for the threshold to reach the upper or lower limit. Hence, the convergence of the selection becomes generally fast, but the selection accuracy becomes worse. Conversely, the higher the number of thresholds, the more likely that the threshold reaches its upper or lower limit through sufficient exploration. Therefore, the convergence needs longer time duration, but the selection accuracy becomes better. In the vicinity of 23 dB in SNR, the BER becomes smaller when the number of threshold steps is reduced; when the number of thresholds is larger than 23 dB in SNR, the BER becomes smaller when the number of thresholds increased. Therefore, it is necessary to set the appropriate number of thresholds depending on the given SNR.

VI. Conclusion

In this paper, we demonstrate an optimization method for user pairing in NOMA systems by using MAB algorithm based on laser chaos on the basis of the bit error rate of the physical layer. The performance of the proposed method is verified by applying MAB algorithm based on laser chaos in the NOMA system. Simulation results show that the proposed algorithm accomplishes a smaller BER than conventional NOMA algorithms. In future work, we will evaluate the performance in more realistic setting problems and implement error correcting codes.

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