

Irregular Repetition Slotted ALOHA Scheme with Multi-Packet Reception in Packet Erasure Channel

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Abstract—In massive machine-type communications (mMTC), a large amount of devices demand to access to the network via a commonly sharing wireless channel. Uncoordinated frequent channel contention associated with channel variation bring a big challenge for improving the performance of massive access system. Irregular repetition slotted ALOHA (IRSA), as a typical grant-free random access protocol, exploits collided signals for packets recovery and possesses a great potential in improving the access capability and throughput of the mMTC system. In this paper, we analyze the performance of IRSA scheme with multi-packet reception where the access point (AP) can simultaneously retrieve multiple packets in a collided signal under packet erasure channel. In particular, decoding failure probabilities during each round of iterative packet recovery are expressed clearly, based on which the packet loss ratio and the throughput of the system are characterized in detail. Simulation results validate the correctness of the theoretical analyses. By selecting appropriate distribution of the replica number of packet, the error floor caused by the packet erasure can be decreased. Besides, the throughput of the massive access system can be improved by optimizing the link load of the IRSA scheme.

Index Terms—Irregular repetition slotted ALOHA, multiple packets reception, packet erasure channel, successive interference cancellation.

I. INTRODUCTION

As one of the most promising applications of 5G, the Internet of Things (IoT) aims to connect billions of devices to completely change our current lifestyle [1]. To meet the massive connection demand of the IoT, massive machine type communication (mMTC) is regarded as one of three application scenarios for 5G [2].

However, in wireless networks where bandwidth is limited and multipath fading exists, it is still a challenge to provide reliable massive access for mMTC [3]. Uplink transmissions from the devices to the access point meet frequent signal collisions, bringing difficulties in improving the performance of massive access system [4]. It is noted that when the number of user equipments (UEs) reaches a certain level, scheduling-based access protocols would cause extremely high access

delay and utility loss due to complex signaling information interaction [5]. In contrast, ALOHA-based access protocol family, allowing UEs sharing wireless resources without specific scheduling, becomes popular in forming grant-free random access system [6].

Traditional ALOHA-based random access schemes, such as slotted ALOHA scheme [7], directly discard collided signals caused by uncoordinated packet transmission. Hence, the channel utility, i.e., the normalized throughput of the random access system is limited and is insufficient to support mMTC [8]. In recent years, some improved ALOHA schemes proposed to make full use of collided signal, promoting the normalized throughput significantly [9]–[12]. For example, to make use of the collided signals, contention resolution diversity slotted ALOHA (CRDSA) was proposed in [9] where a UE would send its packet and a packet replica to different slots. When any packet replica is successfully demodulated, the other replica can be cancelled with the help of the replica position pointer enclosed by the packet. That is, the interference caused by the demodulated packet is removed.

The irregular repetition slotted ALOHA (IRSA) proposed in [10] adopted the similar idea as CRDSA except that a UE can send a random number of replicas of a packet rather than two. In particular, the number of replicas is determined by the corresponding UE according to the probability distribution function, which is the so-called irregular repetition [11]. Moreover, the pointer owned by the packet can find the location of all other packets sent by the same UE [13]. By using successive interference cancellation (SIC) adopted in CRDSA, collision among packets is well resolved and the throughput of the system is enhanced remarkably. It can be seen that CRDSA and IRSA are both simple repetition of the burst, using interference elimination to turn the conflict burst into a treasure. Furthermore, [14] considered the combination of multi-antenna technology and IRSA scheme. Compared with traditional IRSA scheme, this scheme can support greater system link load. It is worth noting that IRSA can also be used in the industrial IoT. The age of information of IRSA protocol was studied for the first time in [15], which proved the potential of modern random access technology in information freshness.

Then, to further improve the successful packets delivery

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probability, coded slotted ALOHA (CSA) was proposed in [12] based on IRSA. Different from the repetition of raw packets in IRSA scheme, the raw packets would be encoded before transmission by using well-designed local packet-level codes in the CSA scheme. It is noticed that the IRSA scheme can be regarded as one typical case of the CSA scheme, where all local coding scheme at devices are repetition coding. Taking the multipath fading and complex interference environment of wireless networks into account, the authors in [16] evaluated the performance of CSA scheme in both packet erasure channels and slot erasure channels.

While the aforementioned works considered that a packet only can be recovered in a slot where no collision signal exists after implementing SIC, advanced communication technologies such as multi-antenna receiver and power capture effect provide for access point (AP) a capable of retrieving multiple packets simultaneously in a collided signal. Motivated by this, the authors in [14] explored the packet loss ratio and the throughput of IRSA scheme in a system where the AP has the multi-packet reception capability. In a recent work, the optimal distribution of the replica number of packet in IRSA scheme have been explored for mMTC system where the AP can retrieve two packets simultaneously [17].

To the best of our knowledge, there is no existing work evaluating the transmission performance of improved ALOHA-based protocol in wireless access systems with multi-packet reception capability and packet erasures. In fact, as the instability of wireless channel is inherent, incidental decoding failure is commonly observed at the AP end even though the multi-packet reception is adopted. While the CSA scheme requires extra computing complexity at all devices, the IRSA scheme avoids the packet level computing at the transmitter side. Motivated by this, we in this work investigate the overall packet loss ratio and throughput performance of the IRSA scheme in a massive wireless access system with multi-packet reception under packet erasure channel. In particular, the incidental decoding failure caused by channel variation is modeled as packet erasure during the transmissions. The main contributions of this work are summarized as follows.

- 1) We derive the decoding error probabilities of the IRSA scheme during the packet recovery process when the AP can demodulate multiple packets simultaneously under packet erasure channel.
- 2) We characterize the packet loss ratio and the throughput in detail. Numerical results show that by selecting appropriate replica distribution of packets, the packet loss ratio floor can be decreased. Moreover, the throughput can be improved by optimizing the link load.

The rest of this paper is organized as follows. In Section II, the system model for IRSA scheme with multiple packets reception capability of AP under packet erasure channel is established. In Section III, we propose the implicit conditions of decoding failure probability during packets recovery when the AP can demodulate multiple packets under packet erasure channel. In Section IV, the system performance is analyzed

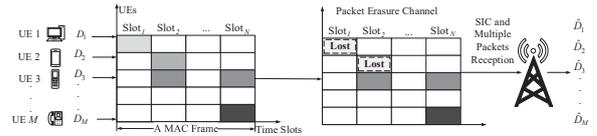


Fig. 1. The schematic diagram of the IRSA-scheme-enabled multi-access system with multi-packet reception under erasure channels. For $i \in \{1, 2, \dots, M\}$, D_i and \hat{D}_i represent the raw data and the recovered data at the AP corresponding to UE i .

under packet erasure channel. Numerical results are presented in Section V. The conclusions are given in Section VI.

II. SYSTEM MODEL

A multi-access system where M UEs attempt to transmit data to an AP by sharing the same wireless medium is considered as shown in Fig. 1. Synchronization is assumed for all the communication patterns. Transmissions are organized into consecutive medium access control (MAC) layer frames where each frame is further divided into N slots. IRSA scheme is adopted at all UEs for countering possible signal collisions and packet recovery failures at the AP, i.e., each UE would repetitively send its packet in multiple slots while the number of packet replicas follows a designed distribution. For clarity, the slot length is assumed to be the same as the packet length, i.e., a packet just fills one slot. Since the wireless channel between UEs and the AP usually experience fading, the successful recovery of the encoded packet from the AP can not be guaranteed even when there is no signal collision in each slot. We model this event as packet erasure and denote the probability of such packet recovery failure by ϵ [16]. By employing multi-antenna technology, the AP is assumed to retrieve up to K multiple packets from each slot. That is, multi-packet reception capability is considered at the AP. To further improve the access efficiency, SIC is adopted at the AP, combating the mutual signal pollution by cancelling the known message from the mixed signals.

A. Iterative Principle of SIC at the AP

Note that it is possible that there are more than K UEs choosing to transmit packets in a particular slot. Hence, even though multi-packet reception is adopted, the AP might fail to recover all the packets transmitted in a particular slot. To further improve the decoding capability, SIC is introduced in the IRSA-based random access framework. Specifically, based on some retrieved decoded packets, the AP tries to recover other packets, reconstruct the signal of the recovered packets, and subtract those signals in the received noisy signal of the corresponding slot the signal being transmitted. This procedure would bring more slots where the AP can retrieve decoded packets by multi-packet reception and can be iteratively operated at the AP until no slot can provide new recovered packets.

To analyze the SIC procedure, graph $\mathcal{G} = (\mathcal{M}, \mathcal{N}, \mathcal{E})$ is usually introduced to characterize the IRSA scheme. Specifically,

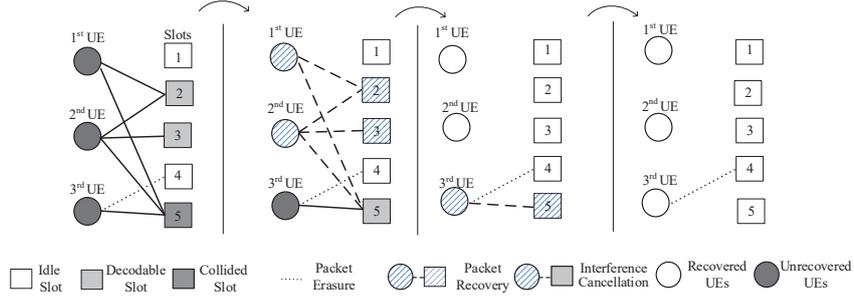


Fig. 2. Bipartite graph representation of the SIC iterative process under packet erasure channel, $K = 2$. In particular, encoded packets transmitted in slot 2 and slot 3 can be recovered in the first iteration because of multiple packets reception ability of the AP.

\mathcal{M} , \mathcal{N} , and \mathcal{E} represent the set of M UE nodes (UNs), the set of N slot nodes (SNs), and the set of edges connecting UNs and SNs, respectively. If i th UE choose to transmit a packet in j th slot, then there is an edge connecting i th UN with j th SN. Fig. 2 presents an example of the SIC procedure where $M = 3$ and $N = 5$. $K = 2$ is considered for the multi-packet reception capability at the AP side. The packet recovery begins from slot 2 and slot 3 using multi-packet reception, which retrieves the raw packets transmitted from 1st and 2nd UEs. According to IRSA scheme, the transmitted signals of 1st and 2nd UEs can be reconstructed, which are used for subtracting known signals from that received in slot 5. As the signals collided in slot reduces, more packets can be recovered from slot 5 and this decoding-subtracting process can be iteratively operated. Finally, all the raw data of the three UEs are recovered. It is noted that one of the packets transmitted by 3rd UE is missed due to packet erasure of slot 4 in the example.

B. Degree Distribution

As shown in Fig. 2, there are edges connected to UNs and SNs in graph \mathcal{G} . Let us define the number of edges connected to a node as node degree. Accordingly, the polynomial representations of UN degree distribution $\Lambda(x)$ and SN degree distribution $\Psi(x)$ can be expressed respectively as

$$\Lambda(x) = \sum_{l=1}^N \Lambda_l x^l, \quad \Psi(x) = \sum_{l=1}^M \Psi_l x^l, \quad (1)$$

where Λ_l represents the probability that the degree of UN is l and Ψ_l represents the probability that the degree of SN is l . From Eq. (1), one can quickly get the average degree of UN $\Lambda'(1) = \sum_{l=1}^N \Lambda_l l$ and the average degree of SN $\Psi'(1) = \sum_{l=1}^M \Psi_l l$. In addition, the degree distributions can also be defined from an edge's perspective as

$$\lambda(x) = \sum_{l=1}^N \lambda_l x^{l-1}, \quad \rho(x) = \sum_{l=1}^M \rho_l x^{l-1}, \quad (2)$$

where λ_l represents the probability that an edge is connected to a UN with node degree l and ρ_l represents the probability that an edge is connected to a SN with node degree l . Then,

the following relations always hold by definition [10]:

$$\lambda(x) = \Lambda'(x)/\Lambda'(1), \quad \rho(x) = \Psi'(x)/\Psi'(1). \quad (3)$$

Recall that UEs randomly select slots to send packets. The probability that a specific UE sends packet in a slot of interest is $\Lambda'(1)/N$. Thus, it has

$$\Psi_l = \binom{M}{l} \left(\frac{\Lambda'(1)}{N} \right)^l \left(1 - \frac{\Lambda'(1)}{N} \right)^{M-l}. \quad (4)$$

Substituting Eq. (4) into Eq. (1), the degree distribution of SN admits that

$$\Psi(x) = \left(1 - \frac{\Lambda'(1)}{N} (1-x) \right)^M. \quad (5)$$

In particular, for large enough M , the number of UEs sending packets in each slot tends to be Poisson distributed. Hence, one has

$$\Psi(x) = \exp(-G\Lambda'(1)(1-x)), \quad (6)$$

$$\rho(x) = \Psi(x)/\Psi'(1) = \exp(-G\Lambda'(1)(1-x)), \quad (7)$$

where

$$G := \frac{M}{N} \quad (8)$$

represents the link load of the system, i.e., the average number of UEs choosing to transmit in each slot.

III. DECODING FAILURE PROBABILITY ANALYSIS FOR PACKETS RECOVERY

Due to the existence of channel erasure, using SIC and multi-packet reception can not guarantee that all the packets transmitted to be recovered in IRSA scheme since it is possible that all the encoded packets are missing. The decoding failure probability of a packets becomes one of the key performance of the CSA scheme. In this section, we analyze the decoding failure probability of the multi-packet-reception-based IRSA scheme under packet erasure channel.

Under the packet erasure channel, some of the packets would be missing at the AP end or equivalently, some of the edges would be erased in \mathcal{G} . We consider an unerased packet transmitted in a slot with degree l in \mathcal{G} . After the packet/edge erasure, the probability that there remains $v - 1$

interfering packets, or equivalently, observing from the corresponding unerased edge, the SN degree reduced from l to v is $\binom{l-1}{v-1}(1-\epsilon)^{v-1}\epsilon^{l-v}$. Accordingly, the SN degree distribution observed from an unerased edge in \mathcal{G} is denoted as

$$\begin{aligned}\tilde{\rho}(x) &:= \sum_{l=1}^M \rho_l \sum_{v=1}^l \binom{l-1}{v-1} (1-\epsilon)^{v-1} \epsilon^{l-v} x^{v-1} \\ &= \rho((1-\epsilon)x + \epsilon).\end{aligned}\quad (9)$$

Recall that in each round of SIC, the iteration starts from the SNs by checking whether packets enclosed in the corresponding interference-cancelled signals can be recovered using multi-packet reception. We say a SN is recovered when the unerased packets transmitted in that slot are recovered, and vice versa. Similarly, we call a UN is recovered if all of its packets are recovered at the AP side or not yet. Randomly selecting an edge in \mathcal{G} , there exists a probability that the connected SN is unrecovered. Let us denote this probability at the end of the i th round of iteration by p_i . Likewise, we denote the probability that the connected UN is unrecovered after the i th round of iteration by q_i . Along with the iteration of SIC, both probabilities would be updated. For example, before the first round of iteration, $q_0 = 1$ while $p_0 < 1$ if there exists some SNs that less than or equal to K packets are transmitted in that slot. Using multi-packet reception, some of the packets would probably be recovered and $q_1 < 1$. Those recovered packets would contribute to the signal cancellation at the SN side and reduce the number of unknown packets in a slot, yielding a p_1 less than p_0 .

Based on the IRSA scheme, unrecovered probability of a SN p_i can be expressed as the following lemma.

Lemma 1. *Consider an AP with multi-packet reception capability K and packet erasure probability ϵ . Given the i th round of iteration of unrecovered probability q_i for the UNs in the IRSA scheme, the unrecovered probability of an SN at the end of the i th iteration is characterised as*

$$\begin{aligned}f(q_i) = p_i &:= 1 - \sum_{k=0}^{K-1} \frac{q_i^k}{k!} \left(-G\Lambda'(1)(1-\epsilon) \right)^k \\ &\times \exp \left(-G\Lambda'(1)(1-\epsilon)q_i \right).\end{aligned}\quad (10)$$

Proof: First, let us analyze the probability that an edge connected to a degree- v SN in \mathcal{G} can not be recovered after the i th iteration of SIC, which we denote it by $p_i^{(v)}$. Specifically, with multi-packet reception capability K , if $v - K$ packets transmitted in a slot have been successfully recovered, the remaining K packets can be recovered. Hence, we have that

$$p_i^{(v)} = 1 - \sum_{k=0}^{K-1} \binom{v-1}{k} (1-q_i)^{v-k-1} q_i^k. \quad (11)$$

Accordingly, the probability of interest p_i can be expressed as

$$p_i = \sum_{l=1}^M \rho_l \sum_{v=1}^l \binom{l-1}{v-1} (1-\epsilon)^{v-1} \epsilon^{l-v} p_i^{(v)}$$

$$\begin{aligned}&= \sum_{l=1}^M \sum_{v=1}^l \binom{l-1}{v-1} (1-\epsilon)^{v-1} \epsilon^{l-v} \rho_l \\ &\times \left(1 - \sum_{k=0}^{K-1} \binom{v-1}{k} (1-q_i)^{v-k-1} q_i^k \right) \\ &= 1 - \sum_{k=0}^{K-1} q_i^k \sum_{l=1}^M \rho_l \sum_{v=1}^l \binom{l-1}{v-1} (1-\epsilon)^{v-1} \epsilon^{l-v} \\ &\times \binom{v-1}{k} (1-q_i)^{v-k-1} \\ &\stackrel{(a)}{=} 1 - \sum_{k=0}^{K-1} \frac{q_i^k}{k!} \tilde{\rho}^{(k)}(1-q_i),\end{aligned}\quad (12)$$

where (a) follows from the first equality of Eq. (9).

On the other hand, it is valuable to note from Eq. (7) and the last equality of Eq. (9) that

$$\tilde{\rho}(x) = \rho((1-\epsilon)x + \epsilon) = \exp \left(-G(1-\epsilon)\Lambda'(1)(1-x) \right). \quad (13)$$

Taking the k -th order derivative of $\tilde{\rho}(x)$, it has that

$$\tilde{\rho}^{(k)}(x) = (G(1-\epsilon)\Lambda'(1))^k \exp \left(-G(1-\epsilon)\Lambda'(1)(1-x) \right). \quad (14)$$

Substituting Eq. (14) into Eq. (12), we can obtain Eq. (10). ■

Eq. (10) manifests the updating process from q_i to p_i in the SIC. According to Lemma 1, it can be seen that the link load G , erasure rate ϵ and multi-packets reception capability K would significantly affect the efficiency of SIC.

On the other hand, unrecovered probability q_i can also be computed as a function of p_{i-1} . In particular, q_i is closely related to the UEs degree distribution. By combining the functions $p_i = f(q_i)$ and $q_i = g(p_{i-1})$, one can characterize the recursion function of unresolved probability of packet erasure channel during the SIC iteration process.

Theorem 1. *The updating process from p_{i-1} to p_i is*

$$\begin{aligned}p_i &= 1 - \sum_{k=0}^{K-1} \frac{[\lambda((1-\epsilon)p_{i-1} + \epsilon)]^k}{k!} \left(G(1-\epsilon)\Lambda'(1) \right)^k \\ &\times \exp \left(-G(1-\epsilon)\Lambda'(1)\lambda((1-\epsilon)p_{i-1} + \epsilon) \right).\end{aligned}\quad (15)$$

Proof: The probability that the desired packet cannot be recovered is equal to the probability that none of the remaining packets sent by the same UE can be recovered. Therefore, the probability that the packets cannot be decoded in the i th iteration can be expressed as

$$\begin{aligned}q_i &:= \sum_{l=1}^N \lambda_l \sum_{j=1}^l \binom{l-1}{j-1} (1-\epsilon)^{j-1} \epsilon^{l-j} p_{i-1}^{j-1} \\ &= \lambda((1-\epsilon)p_{i-1} + \epsilon) = g(p_{i-1}).\end{aligned}\quad (16)$$

By plugging q_i into Eq. (10), the recursion function Eq. (15) follows. ■

Theorem 1 reveals how the distribution $\{\Lambda_l\}$ affects the updating process from p_{i-1} to p_i associated with the effect

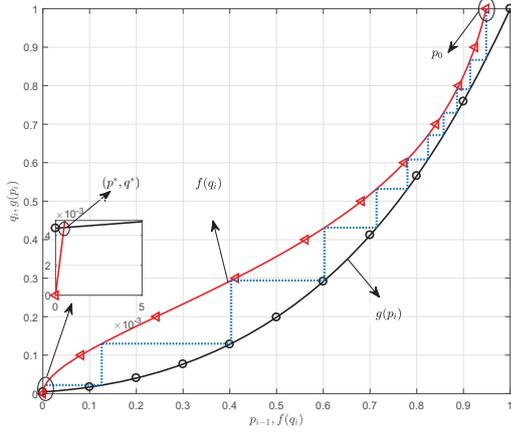


Fig. 3. Iteration of decoding failure probability p_i and q_i . In this case, $\Lambda(x) = 0.5162x^3 + 0.2978x^4 + 0.1287x^5 + 0.0445x^6 + 0.0128x^7$, $\epsilon = 0.1$, and $K = 2$.

of parameters G and K . One can optimize $\{\Lambda_l\}$ and these parameters to reduce the unrecovered probability and improve efficiency of the IRSA system in the packet erasure channel.

With specific parameters, the updating process between p_i and q_i can be visualized based on Eq. (15). For example, let us consider $\Lambda(x) = 0.5162x^3 + 0.2978x^4 + 0.1287x^5 + 0.0445x^6 + 0.0128x^7$ and an AP with multi-packet capability of $K = 2$. We can obtain that the average degree distribution is 3.7399. According to Eq. (8), $G = 1.4$, and $\lambda(x) = 0.4141x^2 + 0.31855x^3 + 0.1721x^4 + 0.0714x^5 + 0.0239x^6$. In this case, we can see that

$$p_i = 1 - \exp(-4.7123\lambda(0.9p_{i-1} + 0.1)) \times (1 + 4.7123\lambda(0.9p_{i-1} + 0.1)). \quad (17)$$

The corresponding iteration process of the decoding failure probability is shown in Fig. 3 as the dotted line while ϵ takes value 0.1. As shown in Fig. 3, for the first round of iterative decoding, $p_0 = 0.9487$, $p_1 = 0.8669$, $p_2 = 0.7909$. When $p_i = p_{i-1} = p^*$, the iteration stops, as marked in Fig. 3. It is noticed that (p^*, q^*) is just the intersection of $f(q_i)$ and $g(p_i)$.

IV. THE PACKET LOSS RATIO AND THE THROUGHPUT

The unrecovered packets contribute to the packet loss of each frame of packet transmission in IRSA scheme. From the Eq. (15), when $p_i = p_{i-1}$, the iteration stops. We define this stopping unrecovered probability as p^* . Note that the unrecovered probability p^* represents the packet unrecovered probability observed from an edge in the graph \mathcal{G} and it is different from the packet loss ratio observed from UN. Let us denote the probability that the AP can not recover all the packets transmitted from a UN by P_{err} . Recall that packet loss occurs in packets transmitted through the packet erasure channel. We consider the UN with l connections without packet erasure in the bipartite graph. Due to the existence of packet erasure channel, the degree of UN would reduce from l to v . Hence, we can obtain the probability p^{*v} which means

the packet loss ratio with the UN of degree v under packet erasure channel. Then it has

$$P_{\text{err}} = \sum_{l=1}^M \Lambda_l \sum_{v=1}^l \binom{l}{v} (1-\epsilon)^v \epsilon^{l-v} p^{*v}. \quad (18)$$

Based on the derived packet loss ratio P_{err} , one can further analyze the throughput of the CSA system, which is defined as the average number of successfully transmitted packets per slot. Denote the throughput by Γ . Recall that there are M UEs in the considered system. Hence, the number of packets successfully transmitted is $sM(1 - P_{\text{err}})$. Further dividing the slot number N of each frame, one can express the throughput of system as

$$\Gamma = \frac{M(1 - P_{\text{err}})}{N}. \quad (19)$$

From Eq. (19), one can find that the system throughput Γ is strongly related to the UE degree distribution $\Lambda(x)$, erasure probability ϵ and the demodulation capability of the AP, i.e., K .

V. SIMULATION RESULTS

Let us present some simulation results to investigate the packet loss ratio and the throughput of the considered system.

Fig. 4 depicts how the packet loss ratio P_{err} varies with the link load G under the packet erasure channel. We set the UEs degree distribution as $\Lambda_1(x) = 0.5162x^2 + 0.2978x^3 + 0.1287x^4 + 0.0445x^5 + 0.0128x^6$ which is suitable for $\epsilon = 0$ and $K = 2$ [17], $\Lambda_2(x) = x\Lambda_1(x)$ and $K = 2$. It can be seen from Fig. 4 that P_{err} increases with the growth of G for all cases where a critical G exists, beyond which P_{err} jumps to approach 1 and the system performance deteriorates dramatically. This is because with the increase of link load, the mixed signals within the system are more difficult to demodulate. For all considered $\Lambda(x)$ and ϵ , there exists a packet loss ratio floor caused by packet erasure. Note that when the link load $G < 1.7$, $\epsilon = 0.1$ with $\Lambda_1(x)$, the packet loss ratio P_{err} is about 10^{-2} . When the link load $G < 1.5$, $\epsilon = 0.1$ with $\Lambda_2(x)$, the packet loss ratio P_{err} is about 10^{-3} . Particularly, for $\Lambda_1(x)$, the error floor decreases while the critical G increases when ϵ increases from 0.1 to 0.2. This implies that for different ϵ , the UE degree distribution should be optimized such that a better error floor and throughput can be achieved. Comparing the packet loss ratio in theory and the simulation results, one can observe that the analytical results well approximate the simulated results for different erasure probability cases. More importantly, one can find from Fig. 4 that when there is no quadratic term of x in the distribution function, i.e., using $\Lambda_1(x)$ and $\Lambda_2(x)$, the error floor caused by packet erasure can be obviously reduced. Hence, by adjusting the degree distribution function, we can alleviate the effect of erasure on P_{err} .

To intuitively observe the effect of different K for different link load G under packet erasure channel, we focus on the curves of throughput which varies with the system parameters such as link load G and K as shown in Fig. 5. In particular, we

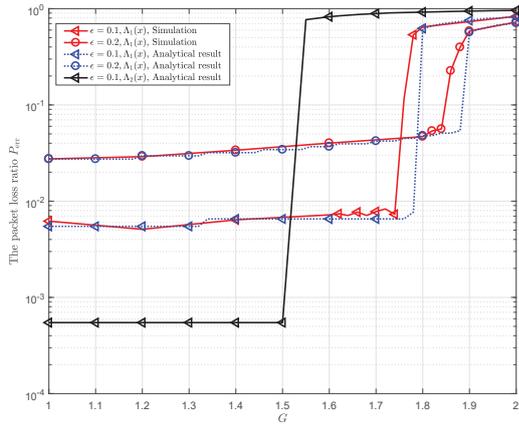


Fig. 4. The packet loss ratio P_{err} v.s. the link load G under packet erasure channel, $K = 2$, $N = 4000$.

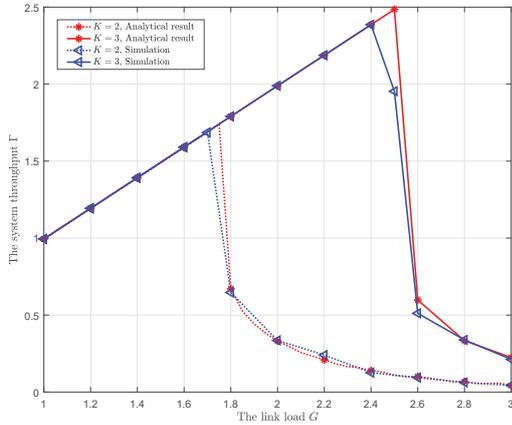


Fig. 5. The system throughput Γ v.s. the link load G under packet erasure channel. $\epsilon = 0.1$, UE degree distribution is $\Lambda_1(x)$, $N = 4000$.

set $\epsilon = 0.1$. One can find that with the increase of link load G , the system throughput would increase from 1. Moreover, the throughput can achieve 2.4 when $K = 3$ and the throughput can only achieve 1.7 when $K = 2$ under packet erasure channel. This shows that throughput can increase as K increases. We can conclude that the capability of AP has a critical impact on the throughput. The better the performance of the AP, the higher throughput of the system can achieve. Moreover, the simulation of system throughput is expressed and we can compare the simulated throughput with the analytical throughput. The simulated throughput well approximates the analytical results under packet erasure channel which validates the correctness of the theoretical analyses.

VI. CONCLUSION

In this paper, we establish a multiple packets reception system model based on IRSA scheme under packet erasure channel. The analysis framework of the IRSA scheme with

the APs multiple packets reception capability is formulated. Then, implicit conditions of the decoding failure probabilities during each round of packet recovery of the SIC process are derived explicitly. Based on the established convergence equation, we characterise the packet loss ratio and throughput in detail. Afterwards, we present numerical results about how the performance varies with the parameters such as ϵ , G , K under packet erasure channel. It is concluded that throughput can be improved by optimizing the link load G . Moreover, some particular UE degree distributions can decrease error floor under erasure channels.

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